

Two-Dimensional Spectral Analysis of Soil Surface Temperature

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ABSTRACT

Soil surface temperature, measured on a 36-by-64 regular grid in a 6-hectare field, is modeled using two-dimensional spectral analysis and a signal-detection procedure. Temperature is found to behave randomly in one direction of the field (width), whereas the distribution in the other direction can be described as a superposition of four cyclic functions.

Investigation of the soil temperature's cyclic behavior revealed that the salt content of the water applied earlier was related spatially to temperature. Different salinities of water had been applied to the field in a cyclic fashion, and co-spectral analysis yielded high correlations at certain frequencies, with higher salinities corresponding to lower temperatures and vice versa.

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THE METHOD INTRODUCED HERE consists of analyzing any spatially varying property as a two-dimensional process such that observations are not independent of each other, but correlate with neighbors in both directions of a two-dimensional space such as a field. As a complement to previous work using one-dimensional state-space models with inferences valid only along a transect, this method allows consideration of variability along both coordinates of a field. We compare the performances of the one- and twodimensional models to show the limitations of the former when variations occur along both axes of the field as in natural conditions. Although soil surface temperature was the object of observation in this study, any other property that varies with space or time could have been selected.

THEORETICAL DISCUSSION

Two-Dimensional Spectral Analysis

A process X(s) is said to be multidimensional. when the parameter *s*, which indexes its values, has several components. This often occurs when a process depends on spatial coordinates as well as on time. If the process is observed and measurements are taken at a fixed time, the model may not involve time at all. For a plane, such as the soil surface, where variability is minor over the depth of interest, the process can be represented by $X(s_1, s_2)$, where s_1 and s_2 are the field coordinates. Conceptually, for a discrete parameter spatial process,

 $s_1, s_2 = 0, \pm 1, \pm 2, \dots$

can be observed at equal intervals. We assume that the two-dimensional process is stationary. That is,

$$E[X(s_1, s_2)] = \mu$$

is constant for all s_1 and s_2 , and the two-dimensional autocovariance function

$$R(m_1, m_2) = E\{[X(s_1 + m_2, s_2 + m_2) - \mu] [X(s_1, s_2) - \mu]\}$$
[1]

depends only on m_1 and m_2 , where (m_1, m_2) is the coordinate of a displacement vector (Shumway 1988).

Suppose that the two-dimensional field is measured at the integer spatial coordinates s_1 and s_2 ($0 \le s_1 \le S_1 - 1$ and $0 \le S_2 \le S_2 - 1$), where there are S_1 values in one direction and S_2 values in the other. The autocovariance function $R(m_1, m_2)$ can be thought of as the average covariance of all observations whose coordinates differ by the vector displacement (m_1, m_2) . The autocorrelation function (ACF) is defined by

$$\rho(m_1, m_2) = \frac{R(m_1, m_2)}{R(0, 0)}.$$
[2]

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We estimate the autocovariance function using

$$\hat{R}(m_1, m_2) = (S_1 S_2)^{-1} \sum_{s_1 = 0}^{S_1 - m_1 + 1} \sum_{s_2 = 0}^{S_2 - m_2 + 1} [X(s_1 + m_1, s_2 + m_2) \quad [3]$$

- \overline{X}] [$X(s_1, s_2) - \overline{X}$],

where $0 \le m_1 \le (S_1 - 1)$, $0 \le m_2 \le (S_2 - 1)$, and \overline{X} is the estimate of the overall mean μ , given by

$$\overline{X} = (S_1 S_2)^{-1} \sum_{s_1}^{S_1} \sum_{s_2}^{S_2} \sum_{s_2}^{X(s_1, s_2)} X(s_1, s_2).$$
[4]

The estimate of the standardized ACF is given by

$$\hat{\rho}(m_1, m_2) = \frac{\hat{R}(m_1, m_2)}{\hat{R}(0, 0)}.$$
[5]

If one assumes that

$$\sum_{m_1=-\infty}^{\infty} \sum_{m_2=-\infty}^{\infty} |R(m_1,m_2)| < \infty,$$

there exists a function,

$$f(\upsilon_1, \upsilon_2) = \sum_{m_1 = -\infty}^{\infty} \sum_{m_2 = -\infty}^{\infty} R(m_1, m_2) \exp\{-2\pi [(\upsilon_1 m_1 + \upsilon_1 m_2)]\}, \quad [6]$$

(Shumway 1988) called the two-dimensional wave-number spectrum. The wave numbers v_1 and v_2 are measured in cycles per unit of distance over the two directions of the random field. The wave-number spectrum measures the variability of the two-dimensional process for each spatial frequency pair (v_1, v_2) .

We estimate the wave-number spectrum as

$$\hat{f}(\upsilon_1, \upsilon_2) = (L_1 L_2)^{-1} \sum_{l_1} \sum_{l_2} |\tilde{X}(k_1 + l_1, k_2 + l_2)|^2,$$
[7]

where $-\frac{1}{2}(L_2-1) \le l_1 \le \frac{1}{2}(L_1-1), -\frac{1}{2}(L_2-1) \le l_2 \le \frac{1}{2}(L_2-1)$, and $\tilde{X}(k_1,k_2)$ is the two-dimensional discrete Fourier transform (DFT) of the process $X(s_1,s_2)$, obtained by applying the one-dimensional DFT successively over both directions.

$$\tilde{X}(k_2,k_2) = (S_1S_2)^{-\frac{1}{2}} \sum_{k_1=0}^{S_1-1} \sum_{k_2=0}^{S_2-1} X(s_1,s_2) \exp[-2\pi i(\upsilon_1s_1+\upsilon_2s_2)],$$

where $v_1 = k_1 / S_1$, and $v_2 = k_2 / S_2$.

The function of $f(v_1, v_2)$ in equation 7 is called the estimated two-dimensional wavenumber spectrum. When the total number of observations is high, the values

$$\chi^2 2L_1 L_2 = \frac{2L_1 L_2 f(\upsilon_1, \upsilon_2)}{f(\upsilon_1, \upsilon_2)}$$
[8]

are approximately independent at different frequencies with a chi-square distribution with $2L_1L_2$ degrees of freedom except at (0,0), (0,1/2), (1/2,0), and (1/2,1/2) where the degrees of freedom are L_1L_2 . This is an extension of the one-dimensional situation, and permits us to detect wave numbers (in both directions) that contribute significantly to the process.

Different smoothing or averaging windows can be obtained by varying L_1 and L_2 in equation 7 in order to remove noise in the wave-number spectrum, leaving only the highest peaks. Their choice is not arbitrary and depends on the number of observations in the data set and experience with similar data. When $L_1 = L_2 = 1$, we have the unsmoothed periodogram.

Three-dimensional plots of $f(v_1, v_2)$ as a function of v_1 and v_2 , for $-\frac{1}{2} < v_1 < \frac{1}{2}$ and $-\frac{1}{2} < v_2 < \frac{1}{2}$ would provide us with the behavior of $X(s_1, s_2)$ as two-dimensional periodic process. As mentioned above, smoothing windows and the chi-square test are available for detecting wave-number pairs that provide significant contributions to the variance of the process.

Signal Detection

If two-dimensional analysis indicates that the wave numbers predominate over one direction of the field, we can describe the process using a one-dimensional model, taking into consideration that we have repetitive measures over one of the directions. Suppose, for instance, that noise is uncorrelated over the direction s_1 and that the predominant periodicities are in the s_2 direction. The appropriate model for detecting the signal in the direction of s_2 (Shumway 1988) is given by

$$X(s_1, s_2) = w(s_2) + v(s_1, s_2),$$
[9]

where $w(s_2)$ is the common signal and $v(s_1, s_2)$ is the noise, assumed to be uncorrelated in the s_1 direction, but stationarily correlated in the s_2 direction.

Note that in equation 9 the noise is in two directions, indicated by s_1 and s_2 . The estimate of the signal $w(s_2)$ is given by the sample mean

$$\overline{X}(s_2) = S_2^{-1} \sum_{s_1 = 0}^{S_1 - 1} X(s_1, s_2),$$

and the one-dimensional DFT of this sample mean is given as

$$\overline{X}(s_2, \mathbf{k}_2) = S_2^{-\frac{1}{2}} \sum_{s_2=0}^{S_2-1} \overline{X}(s_2) \exp\{-2\pi i \upsilon_2 s_2\}.$$

We also define the one-dimensional DFT of the original series as

$$\tilde{X}(s_1, k_2) = S_2^{-\frac{1}{2}} \sum_{s_2=0}^{S_2-1} X(s_1, s_2) \exp\{-2\pi i \upsilon_2 s_2\},$$

where $v_2 = k_2/S_2$ defines the wave-number coordinate in both of the equations with $k_2 = 0, 1, ..., S_2 - 1$.

We can test the hypothesis that the signal is absent rather than arbitrary as a function of wave number υ_2 . This is Shumway's test, and yields the Analysis of Power (ANOPOW) given in table 1. For a type I error (falsely reject the null hypothesis) at a significance level of α , the ratio

$$F_{2,2(S_I-1)} = \frac{\text{Power due to signal}}{\text{Power due to error}}$$
[10]

would be compared with the tabulated value of $F_{2,2(S_1-1)}(1-\alpha)$. Smoothing in a neighborhood of k_2 , say over the interval of $\{k_2+l_2; -\frac{1}{2}(L_2-1) < l_2 < \frac{1}{2}(L_2-1)\}$ introduces $2L_2$ and $2L_2(S_1-1)$ degrees of freedom instead of 2 and $2(S_1-1)$ degrees, respectively.

Source of power variation	Power	Degrees of freedom
Signal	$S_1 \overline{X}(k_2) ^2$	2
Error	$S_{I} = 1$ $\sum_{s_{I}=0}^{\Sigma} \overline{X}(s_{I}, k_{2}) - \overline{X}(k_{2}) ^{2}$	$2(S_{I}-1)$
Total	$\frac{S_{I} - 1}{\sum_{s_{I} = 0}^{\Sigma} \bar{X}(s_{I}, k_{2}) ^{2}}$	2 <i>S</i> ₁

TABLE 1. ANALYSIS OF POWER (ANOPOW)

Input-Output Models

Suppose that in the one-dimensional spatial context, two zero-mean stationary series X(s) and Y(s) might be related by a linear filter model,

$$Y(s) = \sum_{l=-\infty}^{\infty} a(l)X(s-l) + v(s),$$
 [11]

with a(l), $l = 0, \pm 1, \pm 2, ...$ fixed unknown (Shumway 1988), coefficients, and v(s) an unobserved stationary noise process. The sequence a(l) is called the impulse response function of the spatially invariant filter defined by equation 11. We assume here that

$$\sum_{l=-\infty}^{\infty} |a(l)| < \infty$$

The situation in equation 11 may arise whenever the series X(s) and Y(s) represent physical phenomena that are highly related. This is a frequent situation in soil science. For instance, X(s) and Y(s) may represent soil water content and soil water pressure at the identical location. This relationship can be applied to the multiple-input case. If the coefficients a(l) are known, one can determine the behavior of Y(s) based on the realizations of X(s) or vice versa (input-output relationship). In certain instances, measurements of both X(s) and Y(s) are available, and we might be interested in determining whether the two series relate. The filter output given in equation 11 will be known once the impulse response function a(l) and the input process are both known. The impulse response function is estimated from the observed series X(s) so as to minimize the mean square error (MSE) between the output and the linear filtered input, where the MSE is given by

$$MSE = E[Y(s) - \sum_{l=-\infty}^{\infty} a(l)X(s-l)]^2,$$

and we assume that X(s) and Y(s) are stationary, zero-mean series.

The relationship between X(s) and Y(s) as a function of v can also be characterized using the co-spectral properties of the two series, and this allows analysis of the phase response of the linear filter relating the two series. The squared coherence is defined by

$$\gamma_{yx}^{2}(\eta) = \frac{|f_{xy}(\upsilon)|^{2}}{f_{x}(\upsilon)f_{y}(\upsilon)},$$
[12]

where $f_x(\upsilon)$ and $f_y(\upsilon)$ are the one-dimensional spectra of the input and output series, respectively, and $f_{xy}(\upsilon)$ is the cross-spectrum function, defined as the Fourier transform of the cross-covariance function

$$R_{xy}(m) = E[\{X(s+m) - \mu_x\}\{Y(s) - \mu_y\}],$$

say

$$f_{xy}(\upsilon) = \sum_{m=-\infty}^{\infty} R_{xy}(m) \exp(-2\pi i \upsilon m).$$

The coherence can be thought of as the frequency or wave-number dependent correlation between the two series X(s) and Y(s). It gives a measure of how well the input and output series are related and is similar to the ordinary covariance used in classical statistics. For instance, the squared coherence is 1 at all frequencies when Y(s) is an exact, linearly filtered version of X(s) and the noise term in equation 11 is zero.

The impulse response function a(l) can be computed as the Fourier transform,

$$A(\upsilon) = \frac{f_{yx}(\upsilon)}{f_x(\upsilon)},$$
[13]

of the squared coherence. Impulse response functions are estimated using the smoothed sample spectra and cross spectra, defined by

 $\hat{f}_x(\upsilon) = L^{-1} \sum_l |\tilde{X}(k+l)|^2$

and

$$\hat{f}_{yx}(\upsilon) = L^{-1} \sum_{l} \tilde{Y}(k+l) \tilde{X}^*(k+l).$$

where

$$\bar{X}(k) = S^{-\frac{1}{2}} \sum_{s=0}^{S-1} X(s) \exp\{-2\pi i \upsilon s\}$$

and

$$\tilde{Y}(k) = S^{-\frac{1}{2}} \sum_{s=0}^{S-1} Y(s) \exp\{-2\pi i \upsilon s\}$$

are the DFTs of the original sample series with v = k/S, k = 0, 1, ..., S - 1. The statistical method for testing the strength of the linear input-output or regression relation is by comparing

$$\gamma^{2}_{xy}(\upsilon) = \frac{|\hat{f}_{yx}(\upsilon)|^{2}}{\hat{f}_{x}(\upsilon)\hat{f}_{y}(\upsilon)}$$
[14]

with the constant

$$K_a = \frac{C_a}{1 + C_a},\tag{15}$$

where

$$C_a = \frac{2F_{2,2(L-1)(a)}}{2(L-1)},$$
[16]

and α is the type I error for testing $H_0:\gamma^2_{xy}(\upsilon) = 0$ versus the alternate hypothesis that it is significantly different from zero.

MATERIALS AND METHODS

Measurements of soil surface temperature were conducted on a 6-hectare field located near Corcoran, California. The field is part of a large farm but is dealt with as a distinct unit for all agricultural operations. For any management operation, it is considered to be uniform and is treated as such. The deep soil consists mostly of heavy clay (60 to 80 percent) deposited during the Quaternary.

We conducted our measurements in late December following a 3-day period of light precipitation. Water contents at the time of measurements were below field capacity for the top 10-cm layer, and decreased with depth. Temperature of the soil surface was measured by infrared thermometer every 5.18 m (17 ft) in both directions on a 64-by-36-point grid (fig. 1), giving 2,304 points measured. All measurements were made with the thermometer at a 45° angle from the horizontal, 80 cm above the ground.

To account for temperature changes during the time it takes to do one set of measurements of the entire area, we adopted the following method. The measured area consisted of 64 rows running the length of the field and 36 columns running its width. The 36 columns were divided into three groups (I, II, and III), each consisting of 12 adjacent columns, and each group was divided into three subgroups (1, 2, and 3), each



Fig. 1. Schematic diagram of the experimental field, illustrating six salinity levels of irrigation water used respectively on four replicates. Soil surface temperature was measured at each intersection of a row and a column.

with four adjacent columns. Measurements over the entire area were performed six times, alternating the groups in a Latin Square fashion—that is, in all six possible ways of ordering the three groups. Within each group, the subgroups were also treated in a Latin Square fashion. Within each subgroup, the four columns were picked at random. While we made our measurements, we monitored two covariates: soil surface temperature and air temperature near the soil surface. These were measured at fixed locations at both ends of the field before and after measuring a column. The temperature values considered in this study are averages of the six measurements with no correction for the covariates, since we found that two locations differing in temperature by a certain value at a given time did not show the same difference later when their temperatures had changed. In order to record a complete set of measurements in as short a time as possible, we read the measured values into a portable tape recorder. Finally, in order to determine the variability (noise) associated with the measurements, we measured certain columns five times instead of one whenever their turn came for measurement, thus yielding five averages for these specific columns.

RESULTS AND DISCUSSION Analysis by Conventional Statistics

If we assume that all observations are measuring the same parameter (mean soil temperature in this case)—that they are identically distributed and are independent of each other—we can rely on conventional statistics to estimate the parameters of interest with a specified level of accuracy. Conventional statistical analysis is used here only for contrast against two-dimensional spectral analysis, which will be covered in the next section.

Let $f(X,\theta)$ be the probability distribution function of the variable X, where X is the soil surface temperature and is assumed for the time being to be independent of the field coordinates (s_1, s_2) , and θ is a vector whose components are the parameters describing f. Both f and θ are unknown, and we have $x_1, \ldots x_n$ realizations of the variable X. For the present situation, N is equal to 2,304. Our purpose is to find the distribution f and then estimate θ based on the observed values.

We determine the distribution by subjecting the data to a variety of tests. This is equivalent to supposing that f comes from a certain family of distributions, estimating θ for that particular family, and then seeing how well f describes the process X. Normality is the first candidate in most tests, especially when the number of observations is large, as is the case for the present study. Theory provides a variety of tests for normality, and we chose the tests of skewness and kurtosis because of the behavior of the observed sample. Figure 2 shows histograms of the sample and of an equal number of pseudo-random numbers generated from a normal distribution having the same mean and variance as the observed sample, 6.644 and 0.9232, respectively. To generate this pseudo-random sample, we modified the Marsaglia method (coupled generator), using only one generator (Kennedy and Gentle 1980).

From figure 2, we can see that even though the sample yields a bell-shaped histogram, the tails of that curve are more distinct than those of the curve for the generated sample, especially the right tail. Although more temperatures fall in the mean region for the observed sample than for the generated sample, the distributions attenuate at different rates. This phenomenon suggests the presence of some kurtosis in the observed sample. In samples that are very large, deviations from normality are not uncommon. Even though statisticians do not deny that some real data sets are symmetric and have distinct tails, theory has concentrated on "gentler, well-behaving data sets." As noted by Poincarré (1912), "everyone believes in the normal law, the experimenters because they imagine it a mathematical theorem, and the mathematicians because they think it an experimental fact."

To test for departures from normality, we estimated the sample coefficients of skewness and kurtosis (Snedecor and Cochran 1972). We found their values to be 0.047 and 4.498, respectively. The measure of skewness can be positive or negative (for a normal distribution it is equal to zero), and for large samples (n > 150) it is generally distributed with a mean of zero and a variance of $(6/N)^{\frac{1}{2}}$ —0.051 in our situation. For 0.047 to be significantly different from zero, the *p*-value must be between 0.01 and 0.05, not very supportive of the normality of the sample, especially with such a large number of observations. Kurtosis values of less than 3 characterize distributions with flatter than normal tops, whereas values greater than 3 indicate fatter tails. The latter is the case in our situation, as reflected in the shape of the histogram (fig. 2). The sample kurtosis is normally distributed, especially if the sample size is greater than



Fig. 2. Histograms of observed and generated soil surface temperature values.

1,000 (Snedecor and Cochran 1972) with mean 3 and variance $(24/N)^{\frac{1}{2}}-0.102$ in our case. Again, the test rejects the normality hypothesis with a *p*-value less than 0.01. Figure 3 shows a normal probability plot of the sample from which we can see that the non-normality is due exclusively to extreme values on both tails. Unless we wish to look at the world through Cauchy-colored glasses, we must consider these extreme values as outliers and take care of them with a slight trimming. A 5 percent (or less than 5 percent) trimmed mean is a more robust estimator than the usual mean, but only if the data comes from a normal distribution. Another way of bringing these outliers back is by taking logs and performing the normality test on the transformed data.

The normal plot in figure 3 is obtained as follows: If $X_{(1)}, X_{(2)}, \ldots, X_{(N)}$ are the order statistics from the observed sample, then the expected normal value for the rank *j* (out of *N*) is estimated by $\phi^{-1}[(3j - 1)/(3N + 1)]$. That is, the standard normal value corresponds to the probability (3j - 1)/(3N - 1). When the sample comes from a normal distribution, the resulting plot is a straight line.

Another method of estimating the mean temperature and the variance associated with it, if one assumes that observations are independent and identically distributed, is through a one-way analysis of variance model. Neither method above considers the spacings between measurements, so neither is able to provide us with the shortest distance over which to correlate temperature values. The determined sample size does not relate to some specified area to be considered if measurements are to be repeated.



Fig. 3. (a) Normal plot and (b) cumulative plot soil surface temperature observations.

One-Dimensional Spectral Analysis

Figures 4 and 5 show three-dimensional plots of all 2,304 observations viewed from the east and north, respectively. The most striking feature of the plots is that variability is not the same across the field. The observations are noisier in the center, milder in the northern portion, and smoother in the southern portion. This nonstationarity phenomenon cannot be attributed to measurement errors, since all measurements were performed along columns running in the north-south direction, one column at a time, and under the same circumstances. The phenomenon must be innate in the field itself. Another obvious feature is that variability is completely stationary on the east-west axis as compared with that in the other direction. Whether noisy or smooth, the behavior of temperature along any given row is very much the same. A third but much less obvious feature is the probably cyclic behavior on the north-south axis. Temperature is high in the central portion, all across the width of the field, it goes down both to the north and to the south, and then it goes back up at the ends of the measured area. That the data set behaves in a cyclic manner in the north-south direction is not very evident from the three-dimensional plots (fig. 4 and 5).

Plots of temperature distributions for each of the 64 rows and for their respective autocorrelations exhibited different shapes, and tests for detecting signals in individual rows indicated no significant signal of any kind in any of the 64. Although an autoregressive model or a periodic function could be fitted, the fit was statistically poor. Similar plots for the 36 columns revealed a variety of shapes, and even though many plots looked alike, the corresponding estimated autocorrelations indicated different behaviors. Most plots indicated a slight departure from stationarity due to large variances in the central region of the field. Columns 2, 3, 7, 10, and 14, for example, exhibited their highest autocorrelations at lag 7; column 8; at lag 9; columns 1, 5, and 9, at lag 8; column 6, at lag 10; columns 11 and 13, at lag 2; column 12, at lag 4; and so on. In certain cases (columns 4, 7, 8, 9, 17, and 33), the autocorrelation indicated a periodic behavior. In other cases (columns 12, 15, and 20) such behavior is not very obvious. Modeling the columns individually would yield a variety of models. By relying on only a few columns to describe the entire field, we felt we could probably get unrealistic models. Just because a particular model fits a couple of columns does not mean that it can be generalized to the entire field. For instance, if we consider only columns 19, 30, and 34 we will conclude that only white noise exists. For these columns, the autocorrelation at all lags falls well within its 95 percent confidence interval, and the model that we fit may not be statistically strong. The best model is obtained by considering between 20 and 36 columns simultaneously. The fewer the columns considered, the less precise is the description of the average field situation given by the resulting model.

So far, we have only looked at individual rows and columns, a statistically legitimate approach so long as we do not consider averages over either direction. Averaging over one direction can lead to an erroneous model if there is a significant signal in the orthogonal direction. We nonetheless show the average behavior of all columns (fig. 6) and the average behavior of all rows (fig. 7) without model fitting. The autocorrelations of these averages are given in figure 8. As far as the column averages are concerned, we can make the following remarks: (1) all averages show little variability, since all 36 values fall between 6.4 and 6.9°C with the majority around 6.6°C; (2) the variability of these column averages appears to exhibit stationarity; (3) the autocorrelation is well contained in the 95 percent and 90 percent confidence intervals for all lags; and



Fig. 4. Soil surface temperature observations for the experimental field, viewed from the east. Column and row numbers represent increments of 5.18 m.



Fig. 5. Soil surface temperature observations for the experimental field, viewed from the north. Column and row numbers represent increments of 5.18 m.



Fig. 6. Average soil surface temperature versus column number.



Fig. 7. Average soil surface temperature versus row number.



Fig. 8. Autocorrelograms of soil surface temperature for (a) row means and (b) column means.

(4) if we are fitting a model, a harmonic function would be the candidate, given the behavior of the autocorrelation function.

The situation is much different for the row averages. The most striking features of these are their cyclic pattern and the range of temperatures covered, from slightly above 5° to almost 8°C (fig. 9). This periodic behavior was not seen in individual columns, and is only depicted well when about 15 or more columns are averaged. This supports a remark made earlier: that measurements made along one or a few transects do not describe the average situation of the entire field. The autocorrelation of these row averages (fig. 8) strongly supports the harmonic behavior and is well outside the 95 percent confidence interval for many lags. Individual columns were unable to depict periodicity in soil temperature, they were associated with high variability (fig. 9).



Fig. 9. Average soil surface temperature versus row number.

Two-Dimensional Spectral Analysis

When variability of a given variable occurs in more than one direction, the best way to model the variable is by considering a multidimensional stochastic process. In the present study, we have a two-dimensional process $X(s_1, s_2)$: $0 < s_1 < (S_1 - 1)$, and $0 < s_2 < (S_2 - 1)$, with S_1 and S_2 equaling 36 and 64, respectively. The two-dimensional autocorrelation of the process, viewed from different lags in both directions s_1 and s_2 , is given in figure 10. The autocorrelation function plot exhibits striking periodicity along the s_2 direction, relatively high values at lag zero for all rows, and stability in the s_1 direction (from one column to another). The high values at lag zero probably indicate that along the rows, temperature behaves more smoothly than along the columns.

Because the autocorrelation behaves in this fashion, a periodic signal is suitable for describing the process. This signal accomplishes about four cycles along the north-south axis of the measured area. In order to test whether this signal is indeed significant and whether any significant periodicity exists in the east-west direction, we need to examine the two-dimensional periodogram estimating the spectrum. Figure 11 shows such a periodogram for different ranges of the frequencies η_1 and η_2 , where these correspond to the coordinates s_1 and s_2 , respectively. The graph is actually for natural logs of the periodogram, a common way to reduce the range of values covered. The spectrum is estimated with $L_1 = L_2 = 1$. Because of the noise associated with the periodogram, little can be concluded from this graph except for certain peaks for the pairs of frequencies given by $\eta_1 = 0$, $\eta_2 = 0.06$, 0.23, 0.32, 0.38, and 0.44. No major peaks occur when $\eta_1 \neq 0$, so all the variability along the s_1 coordinate is essentially



Fig. 10. Two-dimensional autocorrelogram of soil surface temperature.



Fig. 11. Two-dimensional periodogram of soil surface temperature.

insignificant noise. To eliminate some of the noise associated with the periodogram, we smoothed the latter (fig. 12). In the smoothed version, (1) the only significant signal is in the s_2 direction, since for all values of η_1 the spectrum is flat, and (2) the signal can be described by a linear combination of harmonic functions whose frequencies are the η_2 values corresponding to the significant peaks.

Having concluded that the process $X(s_1, s_2)$ can be described by cyclic functions in the s_2 direction, we now proceed to determine the values of the frequency η_2 , which contribute significantly to the process. This can be tested by the method shown in table 1. The observed values of the F statistic as a function of the frequency η_2 , for L = 1(non-smoothed spectrum) are given in figure 13. The partitionings of the total power into the fraction caused by signal and caused by error are shown in figures 14a and b, from which we can see that four out of the nine peaks in figure 13 are actually caused by signal, and that the remaining five are mostly caused by insignificant noise. We can also see that, once the power due to signal is extracted, the remaining noise power is nearly constant for all values of η_2 . Figures 15a and b show the observed F statistics after smoothing the spectrum with L equal to 3 and 5, respectively. In both cases, smoothing eliminated the minor peaks and left only the four peaks found to be signalcaused in figures 14a and b. The numerator and denominator degrees of freedom of the F statistic for L = 1, 3, and 5 are 2 and 70, 6 and 210, and 10 and 350, respectively. For a 0.01 level of significance, these correspond to 4.90, 2.80, and 2.32, respectively, meaning that all four peaks are significant. By Scheffe's method (Scheffe 1959), the overall level of significance of the tests is 0.04, since we have four peaks, each tested at $\alpha = 0.01$. In other words, the 96 percent confidence bands for these peaks are approximately $\eta = 0.057, 0.24, 0.33,$ and 0.4. This means that the process $X(s_1, s_2)$ can be described by the model



Fig. 12. Smoothed two-dimensional periodogram of soil surface temperature.

$$X(s_1, S_2) = \mu + \sum_{i=1}^{4} [a_i \cos(2\pi\eta_i s_2) + b_i \sin(2\pi\eta_i s_2) + e(s_1, s_2)], \qquad [17]$$

where η_1 , η_2 , η_3 , and η_4 are equal to 0.057, 0.24, 0.33, and 0.40, respectively; a_i and b_i (i = 1, 2, 3, 4) are estimated by

$$\hat{a}_i = S_2^{-1} \sum_{S_2=0}^{S_2-1} \overline{X}(S_2) \cos(2\pi\eta_i s_2)$$
[18]

and

$$\hat{b}_i = S_2^{-1} \sum_{S_2=0}^{S_2-1} \overline{X}(S_2) \sin(2\pi\eta_i s_2),$$
[19]

and

$$\overline{X}(s_2) = S_1^{-1} \sum_{s_1 = 0}^{S_1 - 1} X(s_1, s_2)$$

$$\hat{\mu} = \overline{X} = (S_1 S_2)^{-1} \sum_{s_1 = 0}^{S_1 - 1} \sum_{s_2 = 0}^{S_2 - 1} X(s_1, s_2).$$

That is, the superposition of four combinations of sine and cosine functions constitutes the signal in the s_2 direction, and when associated with the noise $e(s_1, s_2)$, describes the



Fig. 13. Observed F-statistics for the non-smoothed spectrum (L = 1).

process $X(s_1, s_2)$. The coefficients a_i and b_i (i = 1, 2, 3, 4) can also be estimated using the least squares method applied to the model in equation 17. Solutions of equations 18 and 19 yield the \hat{a}_i s and b_i s, and the model in equation 17 becomes

$$X(s_1, s_2) = 6.664 - 0.412 \cos[2\pi(0.057)s_2] + 0.497 \sin[2\pi(0.057)s_2] + 0.176 \cos[2\pi(0.24)s_2] - 0.002 \sin[2\pi(0.24)s_2] - 0.044 \cos[2\pi(0.33)s_2] - 0.004 \sin[2\pi(0.33)s_2] + 0.106 \cos[2\pi(0.4)s_2] - 0.01 \sin[2\pi(0.4)s_2] + e(s_1, s_2)$$
[20]

The performance of the model is shown in figure 16. The model describes the average temperature distribution remarkably well. Absolute deviations from the observed mean values are less than 0.25°C in about 50 percent of the cases and slightly more than 1°C in only two cases. Although the model tends toward slight underestimation of high-temperature values and overestimation of low-temperature values, the mean square deviation is less than 0.25. By inspecting the estimates \hat{a}_i and \hat{b}_i (i = 1, 2, 3, 4), we can see that some of them are very low and can be eliminated from the model. Even though all four peaks in the spectrum were significant, the process can be described by fewer harmonic functions than we previously thought. This is because when we fit one harmonic function, it usually takes care of more than one peak in the spectrum. Another reason is that sometimes only the cosine or sine part of the function is needed to describe the process for a given frequency. The model (equation 20) was reduced to

$$X(s_1, s_2) = 6.644 - 0.412 \cos[2\pi(0.057)s_2] + 0.497 \sin[2\pi(0.057)s_2] + 0.176 \cos[2\pi(0.24)s_2] + 0.106 \cos[2\pi(0.4)s_2] + e(s_1, s_2),$$
[21]



Fig. 14. Signal spectrum compared to (a) total spectrum and (b) error spectrum.



Fig. 15. Observed F-statistic as a function of frequency (smoothed to [a] L = 3 and [b] L = 5).



Fig. 16. Performance of the fitted full model.

and the fit is shown in figure 17. The reduced model still does a good job, though not as good as the full model. In fact, we can use only the frequency that gave the highest peak, and the fit will still be good for practical purposes. That is, a regression through $\cos(2\pi(0.057)s_2)$ and $\sin(2\pi(0.057)s_2)$ yielded a correlation coefficient of slightly less than 60 percent. The frequency that gave the highest peak ($\eta_2 = 0.057$) corresponds to a period of 17.5 points, and all three other periods have values less than this—hence 17.5 points, or about one-quarter of the field in the s_2 direction, is enough to describe the process in the entire field. The sample should not be taken as 18, however, since this result is based on 36 repetitions and does not apply to fewer than 10 to 15 repetitions. If we want to reproduce the field situation accurately, we need at least 12 replicates along the s_2 direction, each of which must consist of 18 observations 17 feet apart. The number of replicates to consider could be determined through a study comparing precision to the cost of measurements.

To find out why the soil temperature fluctuates cyclically in the s_2 direction while it fluctuates randomly in the s_1 direction, we looked at the history of the field over the past few years. It turned out that about 1 year before we made our temperature measurements, six concentrations of salt had been applied to the entire field as part of a long-term study. The concentrations were distributed in four replicates and applied cyclically, as shown in figure 1. The numbers in figure 1 indicate the target salt concentrations of the applied water in grams per liter. A plot of the initially applied salt distribution is superimposed over a plot of the mean temperature across the field in figure 18, and we can see that high temperatures correspond to low salt levels and vice versa.



Fig. 17. Performance of the reduced model.



Fig. 18. Row averages: temperature of soil and salinity of applied water.



Fig. 19. Spectra of temperature (°C) and salt content (g/l).

Measurements of the electrical conductivity of soil extracted after we measured the temperature revealed that the pattern of salt distribution still persists.

To further investigate the relationship between salinity and average soil temperature, we determined the separate spectra for both variables as shown in figure 19. (The spectrum for soil temperature was obtained from the 64 row averages, and should not be confused with the spectrum in fig. 14, which represents the sum of the spectra from the 36 columns. If all columns were exactly the same, the spectrum in fig. 14 would be 36 times higher than the antilogarithm of that in figure 19 for every frequency.) The major peaks discussed above exist in both spectra and for the same frequencies. In figure 19 we can see that even though the magnitudes of the spectra of salt and temperature differ, their shapes are similar. In particular, the highest values occur at the same frequencies in both cases. This is not surprising, considering the common behavior of the two variables (fig. 18).



Fig. 20. Cross spectrum between temperature of soil and salinity of applied water.

We have now shown that temperature could be described by a linear combination of two harmonic functions. For salinity, only the highest peak, that corresponding to the frequency 0.06, is significant—that is, salt distribution can be described by only one harmonic function. This is because salt content follows a more rigorously cyclic pattern than does soil temperature, since the salt was applied as part of a controlled experiment. The period of the function describing salt levels is equal to 17(= 1/0.06), which corresponds to one-fourth of the field—exactly the distance corresponding to one replicate.

Figure 20 plots the cross spectrum between salt content and soil temperature. Recall that the cross spectrum is a measure of association between the two variables at different frequencies. It is near zero everywhere except around $\eta_2 = 0.06$, where it is deeply negative. The negative sign indicates that the two variables are negatively correlated, and the large magnitude indicates that the two variables are highly correlated at the frequency 0.06. We have already seen that both variables can be described exclusively by harmonic functions with this frequency; the cross spectrum reflects the same result. Figure 21 shows the coherence between the two variables. Recall that the coherence is analogous to the covariance in classical statistics and measures the correlation between the two variables for every frequency. We can see that the coherence is high at the frequencies that correspond to simultaneous peaks in the spectra of salt and temperature (fig. 19); that is, the two variables correlate at those particular frequencies. The highest correlation of both variables.

The yardstick for determining which frequencies show significant coherence is given by equations 15 and 16 and yields $K_{0.05} = 0.7$. This eliminates all small peaks, leaving



Fig. 21. Coherence between temperature and salinity.

only that corresponding to $\eta = 0.06$. Again, this corresponds to the observation that both variables can be described well using only this frequency. Because of this result, we can use either variable to estimate the other through the filter given by equation 11. Using salt as the input variable (X) and temperature as the output variable (Y), we estimated the filter coefficients, or impulse response function, a(l) by the method described in the theory. These estimates are shown in figure 22, where we can see that all values are near to zero except when l is near to 32. The peak with the highest magnitude occurs at l = 34, indicating that temperature lags salt content by 1 to 2 points on the average. A lag of two points means that the highest temperature occurs 34 feet behind the lowest salt content. This lag is estimated using all salt levels.

Perhaps temperature lags salt because the salt was applied in strips 51 feet wide, much larger than the lag interval. The lag may also have resulted from the distribution of salt levels as a step function—in fact, this lag is not seen in figure 18, where salt and temperature seem to show no phase shift. All of this supports the hypothesis that salt and temperature vary in opposite directions with no lag or lead. The phase shift may also be realistic, as pointed out by Nielsen, Tillotson, and Vieira (1983), who described how in a field situation a shift may result from microtopography and the angle of the incident radiation. In fact, the shift we have observed is in the north-south direction, exactly as predicted by Nielsen, Tillotson, and Vieira. For the present study, the phase shift for $\eta = 0.059$ is of the order of 4°, which is very small. Another possible explanation is that in the time since salt was applied, water movement on the soil surface and underground may have caused the shift. The fitted filter between X and Y using the results shown in figure 22, is

$$Y(s_2) = \overline{Y} - 0.15[X(s_2 - 2) - \overline{X}] = 1.98 - 0.15 X(s_2 - 2), \qquad [22]$$



Fig. 22. Estimates of the filter coefficients.

where $< s_2 < 63$ and \overline{X} is taken as the averages of the six salt levels applied. This model (equation 22) can be used to predict either variable from the other, provided sufficient measurements of the predictor are used. Since temperature can be measured more easily than salinity, it would be more practical to use temperature as the predictor.

The cyclic behavior of temperature may well result from some other phenomenon not investigated here, and more studies are needed before we can conclusively articulate its relationship with salinity. The only available history of the field besides salinity is texture, but no correlation was found between texture and temperature. Schumgge (1980) found texture to influence microwave emission from the soil, but in the present study the field was texturally uniform. The almost perfect fit between the salt applications and the temperatures prompted us to suspect a cause-effect relationship between the two.

Several hypotheses can be proposed to explain why soil salinity influences temperature. For instance, we would expect a highly saline spot to have a lower water potential and a lower relative humidity than a spot where salinity was relatively low. This difference would cause evaporation rates to differ for the two soils. High-salt soil would tend toward less evaporation, and vice versa. Since our measurements of soil temperature followed the dry season, we would expect salinity at the soil surface to be many times higher than at lower depths as a result of the salt deposition that accompanies the evaporation process. This would result in extremely high salt levels where high salt concentrations were applied (9, 6, and 4.5 grams per liter). Temperature measurements were conducted right after a light precipitation period that followed more than 4 months of dry season. The showers wetted only the top 5 to 10 cm, raising the potential for large differences in the salt concentrations. Another possible reason for the correlation of differences to salinity differences is that soil high in sodium may be dispersed, and as a result that its hydraulic conductivity may be reduced. High sodium is known to reduce hydraulic conductivity as a result of soil dispersion, especially in clayey soils (in the present situation, clay content ranged from 60 to 80 percent). This phenomenon would cause water to infiltrate faster in zones of low salinity than in salty zones, hence the difference in water contents.

Both hypotheses support the association of high water contents with salty spots and vice versa. Because water has a higher heat capacity than the mineral or organic components of soil, water heats much more slowly. For a given amount of energy, water temperature increases much less than do the temperatures of the other fractions of the soil core. Besides, water reflects more energy than dark soil and organic matter, and its temperature remains lower through loss of some energy by evaporation (latent heat). Hence the chain of physical phenomena: higher salt \rightarrow higher water content \rightarrow lower temperature, and vice versa. Morkoc et al. (1985) also found soil temperature and water content to correlate strongly and to vary in opposite directions.

CONCLUSIONS

The present study demonstrates that spectral analysis can be used to (1) detect periodic signals describing two soil physical properties, (2) estimate sample sizes while considering the spacing between the measurements, and (3) relate the two variables as a function of location. We have also seen that, although periodic signals may not seem obvious upon casual observation of a graph, they may provide the only way to describe a phenomenon. We pointed out that, once the sample size is determined using spectral analysis or other techniques, classical statistics can determine the number of replications necessary for a desired level of accuracy. The accuracy level would be determined through an optimization study. Two-dimensional spectral analysis considers variability in both directions of a field, and is thus more realistic. The technique can also be extended to further dimensions in order to include variability as a function of soil depth or time. When applicable, the method is without substitute, since the natural way to represent periodic functions is through sine and cosine functions.

The theory for conducting tests and determining significant components already exists. None of the earlier methods could have described the present situation's cyclic behavior, even though each of them is powerful under appropriate conditions. This leads to the conclusion that, in order to get the best results possible, one should first find out which of the methods is best suited for the data set of interest. This can be accomplished through a general inspection of graphs, tests of independence, multidimensional autocorrelation, and the like. It may be inappropriate to decide on the method of analysis before looking at the data, since each method works best only when the requirements of its use are met. The two-dimensional analysis method has the inconvenience of requiring large, complete, and regular, stationary data sets. However, its use is justified by its lack of substitute and its suitability under conditions that are likely to occur in field situations.

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