HILGARDIA

A JOURNAL OF AGRICULTURAL SCIENCE PUBLISHED BY
THE CALIFORNIA AGRICULTURAL EXPERIMENT STATION

Volume 42, Number 3 • July, 1973

An Economic Model for Selecting Douglas-fir Reforestation Projects

Joseph Buongiorno and Dennis E. Teeguarden

HILGA4 42 (3) 35-120 (1973)

This study presents an economic model for designing and selecting an optimal reforestation program. Given an initial set of resource and policy constraints, the model permits the user to select simultaneously the best set of reforestation projects to execute, the method of reforestation to employ, and the best initial stocking in new plantations. The model was developed for use by managers of large-scale, public land reforestation programs in the Douglas-fir Region of the Western United States.

Results of the study include a package of linked computer programs which provides a completely automated procedure for designing and selecting a reforestation program. The programs have the following functions:

The first program, FOREST, simulates a regime of harvests in plantations consistent with the goal of maximizing present net worth subject to specified constraints. It also generates a set of production functions which describe in terms of present net worth per acre the relationship between number of trees at age ten years and the cost of obtaining that number of trees.

The second program, REGEN II, first calculates the present net worth per acre of alternative reforestation techniques (hand planting, aerial seeding, and natural regeneration) which can be applied to a particular tract of land. Then the same program uses a linear programming model to select the set of tracts, techniques, and initial stocking density which maximizes the present net worth of the whole reforestation program within a set of multiple constraints imposed by available projects, budget, seeds, and seedlings.

A third program, FINAL, lists all projects recommended for reforestation in the current year, method of treatment, seeding density or initial spacing in plantations scheduled for hand planting, and resource requirements.

THE AUTHORS:

Joseph Buongiorno, formerly Research Assistant, School of Forestry and Conservation, Berkeley, is now Forest Economist, Food and Agriculture Organization of the United Nations, Rome, Italy. Dennis E. Teeguarden is Professor of Forestry in the School of Forestry and Conservation, Forester in the Agricultural Experiment Station, and Associate Economist of the Giannini Foundation, Berkeley.

An Economic Model for Selecting Douglas-fir Reforestation Projects^{1, 2}

INTRODUCTION

Purpose of the Study

IN THE DOUGLAS-FIR REGION of the Pacific Northwest, artificial reforestation has increased considerably during the past fifteen years. From 1955 to 1970 the area reforested on public lands alone increased from 20,000 acres to 100,000 acres per year. In 1970 the Forest Service and the Bureau of Land Management, the two main agencies in charge of public forests, spent close to five million dollars for reforestation in the states of Oregon and Washington (figure 1*). Nevertheless, available money limits how much reforestation can be undertaken in any given year and necessitates the creation of guide lines to select those projects which contribute most to the objectives of public agencies.

This study is a result of an attempt to develop new, improved techniques for selecting and designing reforestation projects. It focuses upon the Oregon operations of the Bureau of Land Management (BLM). The study fits into the planning framework introduced in 1965 among federal agencies which requires them to: (1) make explicit the objectives of the programs and relate them carefully to national needs; (2) set out specific plans of work to attain those objectives; (3) analyze and compare the

probable costs and benefits of these plans against those of alternative methods of accomplishing the same results (Fedkiw, 1968).

For the BLM, the reforestation of a tract of land is the result of two years advance planning (figure 2). Planning begins with the forecast of timber harvesting operations occurring two years hence, proceeds with the production of young seedlings in the state nurseries, and concludes with the planting, or seeding, of some chosen tracts of land. (Reforestation of other lands might be postponed because of insufficient resources.) General planning involves the office of the State Director as well as the reforestation staff in each of the Bureau's ten administrative districts in Oregon.

During the planning phase of the study a decision subsystem pertaining to the July–August selection of tracts to reforest was selected for analysis (Teegarden and Von Sperber, 1968). The Roseburg district (figure 3) was selected as the case study area. The district contains some 412,640 acres of Douglas-Fir forests, 4,000 to 9,000 acres of which become available each year for reforestation. In the months of July and August the reforestation staff of the dis-

¹ Submitted for publication June 8, 1972.

² See "Literature Cited" for citations referred to in text by author and date.

* Figures at end of text start on page 68.

trict receives a commitment of funds, seeds and seedlings from the State Director. Given the limited amount of resources and the available land, the reforestation plan which best satisfies BLM goals must be prepared. The purpose of this study was to develop a Reforestation Planning Model (RPM) with which a planner could analyze the alternative combinations of land, budget, and stocks of seeds and seedlings from each of several seed zones so as to maximize the present net worth^a of each year's reforestation program. The model is designed to answer, for each tract of land available for reforestation, the following interrelated questions:

- How many acres should be reforested?
- What site preparation should be applied?
- Is it better to plant or to seed?
- How many trees per acre or pounds of seed per acre should be used?
- From what sources can necessary seeds or seedlings be obtained?

Structure of the Model

To model the Reforestation Planning System we first decomposed the system into its basic elements, then analyzed the several related elements. Finally, the results were synthesized to recompose the whole system in the form of a mathematical model.

The basic decision element relevant to the system is the individual reforestation project. A *project* involves a specific treatment on a tract of land with homogeneous site characteristics. Relevant site characteristics include all factors which influence either costs or prospective benefits: site quality, aspect,

ground cover, and so forth. The first part of the model analyzes the effects of alternative reforestation techniques on the expected present worth of an individual project. The alternative techniques considered are planting and seeding at various densities. Initially it is assumed that all resources, except land, are available in the necessary amount at prices which reflect their relative scarcity. We then construct a model formalizing the competition among alternative projects for limited resources. The model quantifies the results of decisions about each individual project with respect to the performance of the whole system, and attempts to find the set of decisions which optimizes this performance.

The complete Reforestation Planning Model is shown in Figure 4.⁴ It is composed of three independent, yet linked, computer programs designated FOREST, REGEN II and FINAL.

The first program, FOREST, is a stand simulation model with a wide range of applications. When used as the first element of the Reforestation Planning Model, the role of FOREST is to generate production functions showing the relation of the present net worth (PNW) of one acre of land as a function of the initial stocking density and of the establishment cost (figure 5). The calculated PNW corresponds to the optimum rotation with or without a thinning regime. If a thinning regime is required it is the one which maximizes the net revenues per unit of time. Physical yield can be measured in cubic feet or board feet. Further, certain constraints, such as minimum volume of

^a Throughout this study the term *present net worth* or value is intended to mean the values of the outputs and inputs of reforestation projects as determined by using a set of accounting prices which may differ from market prices. The determination of appropriate accounting prices and consistency of present net worth maximization with the BLM goals are discussed below.

⁴ The models developed and discussed in this study are contained in a series of related computer programs written in Fortran IV. In the interest of brevity, the programs have been omitted from this report. However, program listings and data set definitions may be obtained from the School of Forestry and Conservation, University of California at Berkeley, or at the Oregon State Office, Bureau of Land Management, Portland, Oregon.

thinnings and merchantable size of timber, can be imposed in calculating rotation. Production functions are produced by FOREST in the form of a deck of punched cards directly usable as part of the input in the second element of the Reforestation Planning Model, the program REGEN II.

Program REGEN II has four functions, as follows: first the program computes the consequences of alternative decisions regarding the execution of each one of the available projects. For each project the program computes returns, costs, and seed-seedling input requirements corresponding to the planting or seeding of one acre of land with alternative densities. Second, the input-output coefficients obtained as described are used by REGEN II to produce a linear programming model (figure 6) which in turn is used to compute the optimum reforestation program. Third, REGEN II calculates an initial feasible solution to the linear programming problem. This preliminary solution may actually be optimum in cases where a single resource constrains the reforestation program. The preliminary solution is obtained by ranking project alternatives according to three optional criteria: the present net worth; the ratio of present net worth to reforestation costs, (benefit—cost ratio); and the ratio of the present net worth to the seedling or seed input. Fourth, an optional function of REGEN II is to solve the linear programming problem starting from the initial feasible solution.

The optimum solution is produced on punched cards used by the program FINAL to display the reforestation plan.

A Brief Overview

Each element of the Reforestation Planning Model is discussed in detail in the text. A brief enumeration of our central considerations follows.

The theoretical principles on which the model is based are presented, and questions, such as the choice of accounting prices and decision criteria, are discussed. The influence of initial spacing on the output of plantations is analyzed by means of the simulation model called FOREST. Problems of thinning regime and rotation age are also considered. An analysis and synthesis of the reforestation system is then presented. Input-output characteristics of alternative reforestation techniques are derived, and a linear programming model is developed which describes the interaction of all the projects in the district. Program REGEN II is the central feature of this section. The functions of program FINAL are also detailed. Results of applying the model to planning reforestation operations in the Roseburg district in the year 1970 are presented summarily. The final section comments on the operational utility of the model, alternative applications, and its relationship to other parts of the land management system. Some improvements of the model are also suggested.

OBJECTIVES AND CRITERIA FOR PUBLIC REFORESTATION INVESTMENTS

A vast literature exists in the field of capital theory relating to investment criteria. However, no consensus has been reached establishing a theoretically perfect criterion (Little and Mirrlees, 1968).

Most authors seem to agree that the

proper goal in making investment decisions is present net worth maximization (Samuelson, 1964; Radner, 1960; Baumol, 1965). However, there is considerable controversy over questions pertaining to what set of prices should be used to calculate present net worth,

and how projects should be selected in order to maximize present net worth.

The criteria for project evaluation and selection used in the Reforestation Planning Model are not without a certain degree of arbitrariness. But, besides their real theoretical support, they have the quality of laying down all hypotheses in a quantitative, explicit form.

This section first discusses the rationales guiding our choice of criteria. Then the question of choosing a set of prices consistent with the objectives of organizations in charge of reforestation programs is considered briefly.

The Principles

The reforestation system under study here is not a closed one. It has administrative, technical, political, cultural and economic relations with the upper level systems—state, federal government, and society as a whole.

The economic linkages of BLM's reforestation system in one district to the upper level systems are: (1) the objectives, "to manage or dispose of public lands, all in a manner to provide the maximum benefit to the general public" (Bureau of Land Management, 1968); (2) the price system; and (3) the stocks of resources which can be variably allocated throughout the state (seedlings, seed, budget) and throughout the country (budget).

Let us postpone for a moment the complications introduced by the budget and the stocks of seeds and seedlings. Suppose the manager has in hand a set of prices determined for him exogenously by the upper-level system. What decision principle should he observe in order to "provide the maximum benefit to the general public"?

To explain the rationale underlying present net worth calculations we use a

very simple model. This model considers a planning decision involving two periods and one commodity (Samuelson, 1964; Fisher, 1954). The use of one single commodity which we call consumption permits us to abstract from the problem of relative prices of commodities at one point in time. Instead we concentrate upon one price, the interest rate.

The production possibility schedule AB (figure 7) depicts the technology (state of the arts and other resources) available to the economy at some point in time. This technology permits consumers to transform savings (i.e. sacrifice of some current consumption C_0) into future consumption C_1 . This transformation process is what we call investment.⁵ Despite the higher global consumption $C_0'' + C_1''$ secured by investing as much as AC_0'' , society is likely to invest less. The reason for this is the urgency of some of the current needs whose satisfaction requires immediate consumption. The more urgent these needs, the higher the relative value to society of present consumption versus postponed consumption. This relative value or time preference is reflected by the shape of the map of indifference curves I. Along one of these curves, $-dC_1/dC_0$ increases when C_0 decreases. In other words, the smaller the consumption presently available, the greater must be the extra consumption for society to remain at the same level of satisfaction. Assuming that society wants to maximize the utility of its consumption, under a rational planning scheme it will invest the quantity AC_0' . In that way the highest indifference curve is reached by using the available technology. The slope of the productivity curve and of the indifference curve at their point of tangency defines the

⁵ For any investment to be considered at all its yield of future consumption (dC_1) must be at least as large as the sacrifice of present consumption (dC_0) that the investment requires. An investment satisfying this condition is said to have a positive net productivity. Consequently AC is the only relevant part of the production possibility schedule. At point C, $dC_1/dC_0 = -1$ and along the arc BC the net productivity of any investment is negative.

shadow rate of interest in the economy. Thus the shadow rate of interest would result from the optimum allocation of resources over time in the over-all economy. Here the absolute value of this slope is greater than one, corresponding to a positive shadow rate of interest $r = |s| - 1$.⁶ A society producing and consuming at point E, the equilibrium point, is indifferent between consuming or investing an additional unit of current consumption. This we express formally by the equilibrium condition at E:

$$dC_1 = -(1+r) dC_0$$

Let i be an arbitrary rate of interest. We define the present value of the combinations of consumption C_0 and C_1 as the quantity:

$$PV = C_0 + C_1/(1+i)$$

Maximization of present value leads to choosing C_0 and C_1 so that:

$$dC_0 + dC_1/(1+i) = 0$$

$$dC_1 = -dC_0(1+i)$$

Thus, the necessary condition for maximizing present value is identical to the necessary condition for social utility maximization if, and only if, the rate of interest used in the present value calculation is equal to the shadow rate of interest. This result constitutes the rationale for the objective of present value maximization in planning public investment.

The present net worth concept can be generalized to several periods (Hirshleifer, 1958). For a planning period of

T years the present net worth (PNW) of a project is defined as:

$$PNW = \sum_{j=1}^T N_j/(1+r)^j$$

where N_j is the net value generated by the project in year j and r is the shadow interest rate.⁷

It must be emphasized that PNW maximization yields the optimum plan only if the interest rate and the prices used to calculate the net periodic revenues reflect the relative social benefits and costs of the resources used and produced. Once this condition is satisfied, the rule for economic decision is simple: choose among projects the ones which maximize present net worth (Samuelson, 1964).

Criteria for Project Selection

Under a Single Resource Constraint

If no resource limits the execution of a set of projects, the PNW criterion tells us to accept all projects having positive PNW. But when applying this rule to real investment cases, sometimes one or several resources limit the size of the operations so that some projects with positive PNW cannot be executed. Often the amount of money which can be invested in new projects will be such a limiting constraint. What this occurrence shows, of course, is that the limiting resource (capital or other scarce resources) has been improperly priced when the PNW of each project was calculated. But usually we cannot tell what the proper price should be. We are then left with a maximization prob-

⁶ For this shadow rate of interest to be zero two conditions should be satisfied. First, the production possibility curve would have to be symmetric with respect to the 45° line D, which means that all investments in the economy would have zero net productivity. Second, the map of indifference curves should also be symmetric with respect to D which means that society is indifferent between present and future consumption.

⁷ In principle there should be one interest rate r_j for each period. The r_j 's are not necessarily equal since the production possibility schedule shifts over time because of technological and resource availability changes. Furthermore, the indifference map I also varies with the time related changes in social needs. However, as a simplification we will apply the common practice of using one interest rate which must then be viewed as a representative average of the whole series of r_j .

lem under a constraint. How can we go about finding the subset of projects with maximum total PNW given the constraint of one resource?

Suppose first that the resource limiting the dimension of the operations is the budget, M . The PNW of each project is:

$$V_j = B_j - K_j$$

where B_j is the discounted value of all future net revenues, K_j is the initial capital investment to execute the project j . Our problem can then be written:

$$\begin{aligned} \max Z &= \sum_{j=1}^n x_j V_j \\ \text{subject to } \sum_{j=1}^n x_j K_j &\leq M \end{aligned}$$

where n is the number of projects.

$x_j = 0$ if project j is rejected.

$x_j = 1$ if project j is accepted.

We can (after Dantzig, 1963) show this graphically (figure 8). If one coordinate axis measures investment (K_j), and the other measures PNW (V_j), then each project is represented by a point having coordinates (V_j, K_j).

Because of its structure our problem admits an immediate graphical solution. Rotate a ray going through the origin starting from the V axis. Projects corresponding to points successively reached by the ray are selected until the total investment equals the available budget. If fractional solutions are possible (for example reforesting a fraction of a tract) so that x_j can be a fraction, then we obtain an optimum solution. Otherwise, the integer solution

obtained by rounding is sufficiently accurate for most practical problems (Dantzig 1963)⁸

Of course the algorithm consists merely of ranking the projects according to their benefit—cost ratio since:

$$\frac{V_j}{K_j} = \frac{B_j}{K_j} - 1$$

where $\frac{B_j}{K_j}$ is the classical definition of the benefit-cost ratio.⁹

The reasoning above remains valid whatever the type of resource constraining the program (Radner, 1970). We calculate the ratio of the PNW of each project to the input of that resource and rank the project according to that ratio until the resource is exhausted. However, it is necessary that there be one single constraint for the procedure to be valid.

In the Reforestation Planning Model we use the algorithm explained above, in its integer form, to solve the reforestation planning problem in the case where a single resource is constraining, or to determine an initial feasible solution of the problem when there are several resource constraints.

When the budget is the single resource limiting the program, another criterion is sometimes employed to determine the optimum plan. The internal rate of return of a project is defined as the interest rate i which satisfies:

$$\sum_{j=1}^T N_j / (1+i)^j = 0$$

where N_j are the periodic net revenues yielded by the project, and T is the planning horizon.

Under certain assumptions (Eckstein, 1958) the internal rate of return coincides with the benefit—cost ratio,

⁸ An exact solution of the integer problem can be obtained using dynamic or other programming algorithms (Dantzig, 1963; Weingartner, 1965).

⁹ It should be emphasized that the projects are ranked according to their benefit—cost ratio. In no way do we try to maximize the benefit—cost ratio of the program, a procedure which would not lead to the plan of maximum present net worth (Radner, 1960).

and then ranking the projects according to their internal rate of return would yield the plan of maximum present net worth. However, if a resource other than the budget is constraining, the internal rate of return may lead to an improper choice between mutually exclusive projects (Baumol, 1965). Suppose that because of some constraint other than the budget we have to choose between two projects A and B (figure 9). The point of intersection of each one of the PNW schedules with the axis of the interest rate is the respective rate of return of each project. The rate of return criterion would lead to the rejection of project A. However, if r is the shadow rate of interest, maximization of PNW would lead to the rejection of project B. Only if the shadow rate of interest is greater than the rate i_M at which both projects have the same PNW will the internal rate of return criterion give a proper result. Because of this deficiency the internal rate of return criterion is not used in the Reforestation Planning Model.

Algorithm for Project Selection Under Multiple Constraints

In BLM's reforestation work several resources limit the size of the program: the budget, the land available for reforestation, the stocks of seeds and seedlings from each of several seed-source zones. The programming problem referred to when discussing the benefit-cost ratio is then more complex; as a result, more sophisticated algorithms than mere ranking of projects according to some PNW/input ratio have to be used. Later it will be shown that our system can take the form of the maximization of a linear function of activities whose PNW assumes constant returns to scale. Furthermore, all constraints on the system are also linear functions of activity levels so that our

system is a linear program. Then algorithms, such as the simplex method or one of its multiple variations, can be used to solve the optimization problem.¹⁰

In economic terms what the algorithm does is to generate a system of prices (shadow or dual prices) and to simulate a market mechanism which allocates the fixed resources according to the accounting prices. The allocation of resources and the corresponding accounting prices are revised in an iterative fashion so that the objective function is systematically improved. When the optimum program is found the set of accounting prices is such that the accounting losses associated with all the positive activities are non-negative (an economic equilibrium condition) and that the values of the resources is minimized. Furthermore, the accounting price of each fixed resource at the optimum is the value of the marginal product of that resource so that comparison of that accounting price with the price of the resource in the rest of the economic system will show whether it is valuable to decrease or increase the quantity of resource available to the program (Debreu, 1970).

Choosing a System of Accounting Prices

From our theoretical discussion of the shadow rate of interest we have seen that there is no way of determining the proper prices to use in the PNW calculations without solving the complete optimization problem in the whole economy. Since this is impossible, one has to resort to expedients and judgment when trying to determine a proper set of prices.

General equilibrium theory states that under special conditions the market would yield a system of prices corresponding to Pareto optimality, where no person's condition could be made

¹⁰ In principle an integer programming algorithm would be best, but the dimension of the problem renders all the available computer codes insufficient.

better off without worsening the situation of another (Baumol, 1965). But, apart from the fact that those conditions are never satisfied in real markets, Pareto optimality has never been embraced as the goal of any social system. Thus, market prices cannot be the only basis for public project evaluation, although they can be a very useful indication of the forces of supply and demand in the economy (Little and Mirrlees, 1968).

An obvious condition that the set of "accounting prices" (as distinguished from market prices) must fulfill is to be consistent with the value system of the citizens which the public agencies are supposed to serve. These values are commonly expressed through the political process and then allocated in the assignment of a budget to public activities. For example, federal reforestation budgets are determined by Congress through standing policies like the Knutson-Vandenberg Act or by administrative agreement with the O. and C. counties¹¹ in western Oregon which make funds available to the Bureau of Land Management for reforestation (Teeguarden, 1969).

When a certain budget is assigned to reforestation activities, the logical goal of the reforestation program should then be to best use the total amount of the budget. This goal is transmitted down the decision-making ladder to the reforestation manager, who receives a certain amount of money to carry on reforestation projects during the current fiscal year. How should this manager then choose a set of accounting prices consistent with his goal? Two accounting prices are particularly important to the PNW of a reforestation project:

the accounting rate of interest, and the accounting price of future timber outputs. If the interest rate is high and the price of timber low the PNW of many projects on poor sites will be negative. These projects will then be rejected from the plan when PNW maximization is attempted. Consequently the manager may be left with unexpended money. If the accounting prices have been properly determined then a budget surplus indicates that economic resources which have been assigned to reforestation should be used elsewhere in the economy. But if the evaluation of accounting prices is not very rigorous one may well ask whether the rejection of some projects on the basis of the results of project analysis was justified. One reasonable way of getting out of this dilemma is to revise the interest rate downwards and the price of timber outputs upwards until the worst project that the manager can consider in the current year has a barely positive PNW. Then the set of accounting prices so determined will be the basis upon which each project will be assigned a PNW. Finally a programming algorithm will allocate the limited resources to the subset of projects which maximizes total PNW. However, a problem arises because project analysis does not yield any kind of information permitting the determination of the proper size of the budget. Using the scheme described above we may use resources badly needed elsewhere in the economy in extremely poor reforestation projects. Thus accounting prices should not be based upon local requirements only, but they should, as much as possible, reflect the long-run scarcity of resources in the whole economy.¹² If

¹¹ The O. and C. counties are located in lands formerly held by the Oregon and California Railroad Company. These lands, which have been reverted in public ownership by act of Congress, are now administered by the Bureau of Land Management.

¹² The fact that it is the *long-run* scarcity which is relevant is important to note. Because the budget is small this year is not a valid reason to increase the accounting rate of interest to evaluate the PNW of projects which will be managed for fifty to ninety years. Doing this would unjustifiably give advantage to projects and management alternatives yielding returns quickly.

project analysis is to serve the purpose of helping to allocate limited resources rationally then the PNW of all projects must be calculated upon a set of accounting prices determined by higher authority in the decision-making system. The results of project analysis, together with other information, can

then be used as feedback to periodically revise the set of accounting prices. A step in that direction has been taken by the federal government, recommending that all federal agencies use a common rate of interest.¹⁸ The procedure should be generalized to other prices as well.

A SIMULATION MODEL FOR ESTIMATING DOUGLAS-FIR YIELDS

Theoretical Foundations of the Model

Present worth maximization is used in RPM as the fundamental guide to making plantation investment decisions. In this section, the principle will be applied to the problem of deriving estimates of future plantation yields, which in turn depend on prospective management programs. The management decision of central importance concerns initial stocking densities in plantations. This decision is important for two reasons. First, the outputs of a plantation depend upon the initial density. Second, the cost of establishing a plantation is a function of initial density. The decision is even more important if the supply of seeds and seedlings limits the scale of the reforestation program. However, the initial density decision cannot be dissociated from other management decisions because the present worth of a plantation also depends on future management inputs: thinning, protection, fertilization, and so forth. Two management decisions are considered in FOREST besides initial density: first, the schedule of intermediate cuttings as the plantation matures, and second, the age at which the plantation is to be replaced by a new one.

Intermediate cuttings. The role of intermediate cuttings is to insure the best use of the site and growing stock throughout the life of the plantation. Crowding of trees inhibits their growth while understocking does not make full utilization of the site. Investment theory may be used to determine the proper stocking level. To illustrate, consider the growth of a stand in a particular year (year one in figure 10).

For each value of residual stock following harvest there is a corresponding value of the stock at the end of year one. This relationship is represented by the production possibility schedule, which describes the alternatives available to the manager. Note that the first quadrant in figure 10 is the exact equivalent of figure 7 showing the optimum investment policy for a two-period investment decision. Indeed the thinning decision is an investment decision. It tries to answer the question: How much harvest of timber should be sacrificed this year in order to increase stocking next year? And the answer is derived in the manner described earlier: maximize PNW. Assume the alternative rate of return i is given to the decision-maker; the goal of PNW maximization in the present case can then be written:

$$\max(-S1 + S1'/(1+i))$$

¹⁸ Budget Bureau Circular A-47 recommends: "... Where the economically useful life of the project is expected to be longer than 15 years, the rate of interest shall be calculated at a rate based upon the average of interest payable by the treasury on obligations, if any, outstanding at the end of the fiscal year preceding such computations, which upon original issue, had terms to maturity of 15 years or more."

Where $S1$ is the value of the residual stock following harvest, and $S1'$ is the value of the stock at the end of the year.

The above expression corresponds to maximizing the net value growth of the stand, as recommended by other writers (Duerr, 1960; Chappelle and Nelson, 1964), since:

$$\max(-S1 + S1'/(1+i))$$

is equivalent to:

$$\max((S1'-S1)-iS1)$$

The necessary condition for a maximum is:

$$d(S1'-S1)/dS1 = i$$

That is to say, the stand should be thinned up to the point where the prospective marginal value growth per cent of the stand is equal to the interest rate.¹⁴

The pattern of stock accumulation during three successive years is described in figure 10. The production possibility schedules show the trade-off between the harvest value in one year and the stock value in the following year. For example, if all the stock in year one is harvested, then the value of the stock at the end of the year is 0. If, on the contrary, nothing is harvested in year one, the value of the stock at the end of the year is $S1$. The shape of the production possibility schedule reflects the diminishing marginal productivity of the stock at a given point in time when the stock increases. The ideal thinning in year one harvests the amount $H1$ thus leaving the residual stock $S1$ which grows into the stock $S1'$ in year two. The marginal productivity of the optimum residual stock is always equal to the interest rate i .

¹⁴ Different rules involving maximization principle of dynamic programming have been applied to the problem of optimum stocking determination (Naslund, 1969; Amidon and Akin, 1968). However, the application of these rules to real cases has not given answers contrary to the one produced by the principle exposed here (Amidon and Akin, 1968).

In practice, the volume of thinnings extracted at a certain date must meet some other silvicultural constraints (which must figure in the model) in order for the thinning to be operationally feasible. The maximization of the net value growth of the stand under such a constraint may then lead to a residual stock whose marginal value growth is very different from the interest rate.

Rotation age determination. If a certain stand of timber is maintained on the land over time the benefits from growing a new plantation on the same land are postponed. To determine the optimum replacement age (or rotation age) of an even-aged plantation, it is necessary to determine the age at which the final harvest of the stand should occur in order to maximize the present value of a perpetual succession of plantations. Given a certain initial stocking of a plantation, the rule of thinning outlined above will yield a stream of net revenues:

$$N_0, N_1, \dots, N_t$$

The PNW of this stream of net revenues during a rotation of t years is given by:

$$PNW_t = \sum_{j=0}^t N_j/(1+i)^j$$

where i is the interest rate. Then, the PNW of a perpetual succession of rotations on the same area is:

$$PNW_0 = PNW_t + PNW_t/[(1+i)^t - 1]$$

The value of t , t_0 , which maximizes PNW_0 is the optimum rotation age (Chislom, 1965; Chappelle and Nelson, 1964). This value is sometimes called the "soil rent" (Duerr, 1960). It is easy

to see from the formula of PNW_0 that t_0 depends upon all the factors, including the establishment cost, which influence PNW_t . Again, other management constraints may impose rotations quite different from the economic rotation and the model should reflect such constraints.

Initial stocking. Given these principles of thinning and rotation age determination, it is possible to determine the PNW of a plantation for a certain initial stocking and the cost of obtaining that initial stocking. If the initial stocking and the establishment cost are varied, we can obtain a value production function like the one shown in figure 11. This production function may then be used as a basis for choosing between reforestation alternatives.

Suppose, for example, that we consider two alternatives to reforest a tract of land on site 90. Alternative one is natural regeneration which will cost nothing, and is expected to yield 100 crop trees per acre at age 10. Alternative two consists of planting 680 seedlings which will cost 50 dollars per acre and yields 400 crop trees at age 10. From the production functions in figure 11 we find that alternative one has a PNW of 360 dollars per acre while alternative two has a PNW of 480 dollars per acre and thus would be preferred.

The following sections present a computer model, **FOREST**, which generates value production functions such as illustrated in figure 11.

The Model Forest

FOREST is a simulation model composed of two basic elements. The first element simulates the growth process in an even-aged stand of Douglas-fir trees. The second element models the decision-making system (thinning regime and rotation age), according to the principles discussed above, for purposes of

deriving yield estimates for managed Douglas-fir plantations.

Simulation of tree growth and mortality. The basic tool required to derive plantation yield schedules is a model of stand growth which predicts response to alternative initial stocking density levels. Of the theories of stand growth (Assman, 1953; Möller, 1954), Möller's hypothesis has been the most widely used to derive thinning schedules for Douglas-fir (see Staebler, 1960; Hoyer, 1966). Möller's theory simply states that beyond the "point of full occupancy" of the site, stand growth is constant. However, Möller's theory is imprecise and has been contradicted by recent growth studies of Douglas-fir stands (Curtis, 1967). Further, such growth studies as are available are inadequate for purposes of this study, in part because factors important to management evaluation (number of trees, diameter distribution, density effect) have not been considered; accordingly, a simulation model was developed to estimate stand growth in relation to initial stocking and other management practices. The model predicts tree growth as a function of tree characteristics and the tree's physical environment. Having described single tree behavior, the growth of a stand of trees can then be simulated and the response of the stand to management analyzed. This approach was first explored by Newnham (1964). However, Newnham's model uses arbitrarily derived mortality and growth functions for the single tree which cannot be estimated by field measurements. The present model avoids that defect and differs from Newnham's model in other respects. But the fundamental idea of using the single tree as the basic element of stand growth analysis is the same.

A simplified representation of that part of **FOREST** which simulates stand growth is given in figure 12. First,

stand data are read by the main program. Then the subroutine INITIA generates the forest stand at its initial state. A stand is represented by a population of trees distributed over a square "working grid." Before tree growth is simulated, the subroutine EXPAND borders the working grid with eight grids identical to the working grid. This is done to avoid edge effects. Then for each tree in the working grid the following three steps are taken. First, the environment of the tree is analyzed by the subroutine SEARCH. Second, the subroutine MORTAL predicts the death or survival of the tree during the current time interval. Third, if the tree is not declared dead, the subroutine GROWTH predicts its growth. These three steps are repeated for each tree on the working grid. Finally, stand characteristics are updated and displayed by the subroutine OUTPUT. Then all processes, starting with the subroutine EXPAND, are repeated until the simulated age limit is reached. Following sections describe in more detail each function of the stand growth model.

Initial stand representation. The program subroutine INITIA models the stand under consideration at its initial age. A flow chart of the operations performed by INITIA is given in figure 13. The initial stand is represented by a population of trees distributed upon a square working grid. The distance between each node on the grid is defined by the user. The coordinates of each node are defined by two integer numbers, *I*, *J*. Given the maximum number of nodes on each side of the grid and the internode, the area of the working grid can be calculated. Then, given the initial number of trees per acre in the stand, the probability of not

having a tree at one node (PDEAD)¹⁵ can be calculated if a random distribution of trees is assumed. Comparison of PDEAD with a random number between 0 and 1 decides whether a tree should be represented at a particular node of the working grid. By properly choosing parameters, this procedure permits generating a regular or random distribution of trees on the grid. If a tree is to be present at a particular node, its diameter¹⁶ is drawn from a normal distribution whose mean and standard deviation are defined by the user. Otherwise the "tree" is given a diameter of 0 inches.

When the diameter of a tree has been calculated, INITIA proceeds to determine the height, commercial volume, value, and diameter class of the tree. A function subprogram, HEIGHT, calculates the height of the tree in feet, as a function of its diameter (*DIAM*) using the function (Meyer's Function, cited by Ker and Smith, 1955);

$$\text{HEIGHT} = 123.5 \\ - 119 \exp(-.11 + \text{DIAM})$$

The subprogram *CFT* calculates the commercial volume of a tree in cubic feet or board feet, as a function of its diameter (*DIAM*), height (*HGT*), and minimum diameter of merchantable trees (*DLIM*) as follows:

$$\text{if } \text{DIAM} \leq \text{DLIM}, \text{ then } \text{CFT} = 0$$

Otherwise, in cubic feet (Curtis, 1967):

$$\text{CFT} = .00409 \text{ DIAM}^{1.835} \text{ HGT}^{.957}$$

or, in board feet:

$$\text{CFT} = \exp(-6.975) \text{ DIAM}^{1.73} \text{ HGT}^{1.643}$$

This last function was derived for this study using published data (McArdle *et al.*, 1961).

¹⁵ All variables, constants and function names in the text correspond to the FORTRAN names used in the programs.

¹⁶ All diameters referred to are measured breast height, in inches.

The function VAL calculates the value of one unit of volume as a function of tree diameter and of the stump-age price (ALPH):

$$VAL = ALPH \times DIAM$$

This value is decreased by a constant DELP when thinnings are evaluated to take account of the higher cost of harvesting thinnings (figure 14).

Finally, the subprogram IDCL determines the diameter class of a tree. The total stand is divided into one-inch diameter classes to give a picture of the diameter distribution in the stand.

When each characteristic of a tree at a particular node has been determined, the subroutine INITIA proceeds to update the number, value, and diameter distribution of the trees on the working grid. The sequence of procedures, starting with the determination of the presence of one tree at a particular node, is repeated for each node on the working grid. Then INITIA transforms the data for the working grid into per acre data and displays the initial stand characteristics (see figure 24b).

Tree environment analysis. Because the trees at the center of the working grid are completely surrounded by trees while those on the edges of the grid are not, the former would exhibit a behavior completely different from the latter. To avoid such "edge effects" the working grid is bordered by eight grids identical to the working grid. The bordering is accomplished by the subroutine EXPAND (figure 15) everytime the working grid has been modified due to stand growth, mortality, or harvest of trees.

In FOREST, three environmental factors influence tree growth and the probability of death during a simulated time interval: (1) the *site quality*, as measured by site index (McArdle *et al.*, 1961), which is fixed once the initial condition of the stand has been set; (2)

the *point density* around the central tree (Spurr, 1962); and (3) the *crown class* of the central tree with respect to its neighbors. To determine point density and crown class a sample of eight trees is taken around the central tree. This sampling is done systematically by subroutine SEARCH, which observes each quadrant around the central tree (figure 16). Once the sampling has been performed, the average diameter (AVDIA) of trees in the sample plus the central tree is calculated. Then SEARCH notes the height (HDOM) of the dominant tree in the same group. Finally, the point density around the central tree is calculated by the function subprogram BASAL on the basis of Spurr's formula (1962):

BASAL =

$$75.655 \sum_{n=1}^n \frac{[(n + \frac{1}{2}) (D_n/L_n)^2]}{8}$$

where BASAL is the point density in square feet per acre; n is the rank of the ratio D_n/L_n when the eight ratios for each tree in the sample are ranked in decreasing order; D_n is the diameter in inches of the n^{th} tree; L_n is the distance in feet of the n^{th} tree to the central tree.

Tree mortality model. Mortality is here defined as "normal mortality" which occurs in Douglas-fir stands when no catastrophic condition prevails. Staebler (1960) estimated normal mortality in permanent sample plots aged 26 to 93 years. The predicting equations he derived are used in FOREST to forecast the probability of death of a particular tree during a time interval of DAG years. For dominant and codominant trees, the equation is:

$$PD = (.00496 + .00008 AG - .00041 DBC) DAG \\ (R = 266)$$

and for suppressed and intermediate trees:

$$PS = (.013 + .00054 SI + .00061 AG - .00783 DBC) DAG \\ (R = .75)$$

where

SI = site index in feet at 100 years

AG = tree age in years

DBC = tree diameter in inches

DAG = time interval in years.

In the model these results are interpreted and used as follows by the subroutine MORTAL and the function $PDTH$ (figure 17). A tree is classified as dominant and given the probability of death $PDTH = PD$ if the point density around that tree is less than or equal to the basal area of an open grown stand of average diameter, $AVDIA$. The basal area of open grown stand (BAOP) is calculated using the following function:

$$BAOP = AVDIA^{.88} \exp(.23 - .026 AVDIA)$$

which was derived for this study using published data (Smith *et al.*, 1961). If the point density is greater than BAOP there is competition around the tree currently analyzed. In that case the tree height (HBC) is compared to the height of the dominant tree ($HDOM$) in the group defined in the preceding paragraph. If $HBC \leq [HS = (2/3) HDOM]$, then the tree is classified suppressed and given the probability of death $PDTH = PS$. Otherwise the tree may be intermediate, codominant or dominant, and its probability of death is a weighted average of PD and PS :

$$PDTH = PD + [(HBC - HDOM) / (HDOM - HS)] (PD - PS)$$

Comparison of $PDTH$ with a random number between 0 and 1 decides, then,

whether the tree in consideration will die during the current interval of time DAG . If the answer is yes, the subroutine MORTAL updates the data relating to the volume, value, and diameter distribution of the mortality during the simulated time interval DAG . Then control is returned to the main program to calculate the growth of the living tree.

The tree growth model. If the tree currently analyzed is not dead at the outset of the current period of time, then the subroutine GROWTH proceeds to forecast the growth of the tree. The essential step here is the forecast of the diameter growth. A good growth function should relate diameter growth to the individual characteristics of the tree and its environment. Individual tree characteristics should include the age, diameter, and crown class of the tree. Environmental characteristics should include competition from nearby trees and site quality. Although the model FOREST has been designed to easily incorporate such a function, it was beyond the scope of the study to obtain data by direct field studies. Instead, the available yield functions were used. These yield functions describe the behavior of a stand of trees rather than any individual tree. In the present case, the stand of interest is the group of trees surrounding and including the one being analyzed by the model. Such a group of trees is sampled and measured by the subroutine SEARCH. Its basal area is the point density (BAR), its average diameter is $AVDIA$. The assumption made here is that the tree analyzed grows like the tree of average diameter in the group. Two sources of data were used to predict the growth of this tree: normal yield tables for Douglas-fir (McArdle *et al.*, 1961) and a Douglas-fir stand model developed by the Bureau of Land Management (1965). The assumption made is that the normal yield tables give the minimum diameter growth which a stand

of trees of given basal area can achieve.¹⁷ The BLM model, on the other hand, gives the upper bound of the diameter growth that a stand of given basal area can achieve.¹⁸ Between the two bounds, diameter growth (DD) of the average tree in a stand of given basal area is computed as the weighted average of the bounding diameter growth as follows (figure 18):

$$DD = [(AVDIA - DM) (DDX - DDM) / (DX - DM)] + DDM$$

where:

DM is the average diameter of a normal stand of basal area BAR.

DX is the average diameter of a stand of basal area BAR according to BLM data.

DDM is the diameter growth of the trees in the normal stand of basal area BAR.

DDX is the diameter growth of the trees in the BLM model when the basal area is BAR.¹⁹

The list of functions used by the function subprogram DGTH to calculate the diameter of the tree currently analyzed is given in table 1. When this diameter growth is returned to the subroutine GROWTH the new diameter, height, volume, and value of the tree are calculated.

Each procedure starting with the subroutine SEARCH and ending with the subroutine GROWTH is repeated for each living tree on the working grid. Then data describing mortality and the

current stand situation are updated and displayed by the subroutine OUTPUT after transformation into per acre data (see figure 24c).

Simulation Results

Growth forecasts based on the model were compared with data available from other studies. Little work describing the effect of alternative initial spacings upon the productivity of plantations has been done. In the Pacific Northwest, data are available from the Wind River plantation tests (Reukema, 1970), but these plantations are relatively young (40 years), relate to a single site index (110), and represent a limited sample. Other data have been reported by Duff (1956) for Douglas-fir plantations in New Zealand, based on stands 40 years old or less. Growth predictions obtained with the model FOREST are compared to Reukema's and Duff's estimates in figures 19a-19d. It appears that the model gives a very good picture of survival beyond age 10 in plantations at alternative densities. It also gives a good picture of diameter growth at different spacings, although the results of the model are higher for close spacings compared to the Wind River plantations. The results compare less well in terms of basal area and cubic feet growth, but because of the scarcity of empirical data, no definitive conclusion can be drawn.

Further verification of the growth functions would be necessary to improve the model. However, for subsequent growth predictions obtained with FOR-

¹⁷ The normal yield tables give the diameter growth of dense stands (1460 stems on site 140 at age 20) and thus it is unlikely that current plantations will have lower diameter growth for stands of the same basal area.

¹⁸ When plotted against age, the diameter of the average tree in the BLM model is slightly above the diameter of open grown trees as measured by Newnham (1964). Thus it is unlikely that greater diameter growth be achieved in stands of the same basal area.

¹⁹ Due to the limitations of the available data, it was necessary to discard age as a factor influencing diameter growth. This seems acceptable if one considers that the diameter growth of open-grown trees is about constant during the first decades of tree life (Newnham, 1964). Decrease in diameter growth through age shown by the ordinary yield tables may be due to tree competition in the stand rather than to age.

TABLE 1
FUNCTIONS USED BY THE FUNCTION SUBPROGRAM DGTH TO CALCULATE THE DIAMETER GROWTH OF A TREE

SITE INDEX	NORMAL STAND		B.L.M. MODEL	
	Average diameter (DM) when basal area is BA.	Diameter growth (DDM) when basal area is BA and average diameter is DM	Average diameter (DX) when basal area is BA	Diameter growth (DDX) when area is BA and average diameter is DX
180 or more	$.73 + .0138BA + .00023BA^2$	$.45 - .00082BA$	$.31 + .059BA + .0003BA^2$	$.75 - .000184BA$
150-189	$.55 + .0104BA + .00019BA^2$	$.4 - .00081BA$	$.25 + .054BA + .00027BA^2$	$.64 - .00163BA$
120-149	$.29 + .0115BA + .00017BA^2$	$.28 - .00058BA$	$.11 + .057BA + .00024BA^2$	$.55 - .0015BA$
100-119	$.087 + .0091BA + .00018BA^2$	$.23 - .00057BA$	$.008 + .06BA + .00027BA^2$	$.39 - .00112BA$
99 or less	$.03 + .0039BA + .00023BA^2$	$.174 - .00053BA$	$-.02 + .062BA + .00053BA^2$	$.03 - .00144BA$

EST, the growth model is used in the Reforestation Planning Model.

Design of the Decision-Making System

Decisions regarding the management of a stand, given the initial density, concern the thinning regime and the rotation age. The theoretical principles governing these decisions were given above. The following sections describe how these principles are combined with the growth simulation system for purposes of estimating yields in the model FOREST (figure 20).

The thinning algorithm. Once the characteristics of the stand at a certain point in time have been determined, the stand is thinned if thinning is specified by the user and if other conditions are satisfied. In particular, the stand must have a commercial volume at least equal to the minimum thinning level acceptable, and the number of growing trees must be above the user-specified minimum. If the thinning conditions are satisfied and if no thinning has been previously attempted at that point in time the current state of the working grid and the mortality data are stored in memory by the subroutine STORE. This permits the stand to be reinstalled in its prior-to-thinning state if a prospective thinning fails to pass the test of increasing net revenue per unit of time. The thinning of the stand is performed in several successive trials by the subroutine THIN (figure 21). Only the trees of commercial value are considered for thinning. On the first thinning trial all trees which are predicted to die during the current period are harvested. On each of the subsequent thinning trials, only one tree is harvested. However, this operation is executed only if the thinning up to this point is acceptable, i.e. if it leaves enough trees and if the percentage of stock harvested is above a certain limit. The bounds of these constraints are specified by the user. The

chosen tree is the one with minimum value growth. Every time a tree is harvested the growth model describes the reaction of the stand during the current time period. The thinning is continued until the marginal value growth per cent response of the stand to the withdrawal of one tree is just larger than the rate of interest (figure 22). However, the thinning can be continued until the minimum thinning volume is obtained, if this improves the net revenue per unit of time. If the thinning fails to increase net revenue, the working grid is reinstalled in its prior-to-thinning state by the subroutine UNSTORE. The description of the thinnings, number of trees, diameter distribution, average diameter, volume and value, is displayed by the subroutine OUTPUT. At that time the PNW of the cumulative past thinnings plus the PNW of the current stand is calculated and displayed (figure 24d). The type of thinning obtained by the algorithm would, under specific assumptions regarding benefits and costs, maintain an average stocking level of about 4000 cubic feet in a plantation on site 110 with 680 trees per acre at age 10 (figure 23a). This thinning regime would increase by almost 100 per cent the PNW of a plantation (figure 23b). However, the age of maximum PNW would be delayed by about forty years. This longer time of production is taken into account in evaluating reforestation alternatives by the part of FOREST which we proceed to describe.

Estimation of present net worth. The subroutine EVAL calculates the optimum rotation and the corresponding PNW of a plantation according to the principles discussed earlier in this section. The calculations are performed for a user-defined set of establishment costs

which are the costs of obtaining the initial density of the stand being analyzed. A minimum rotation age can be specified by the user so that if the optimum rotation length calculated is less than the minimum required the PNW will be determined for the specified rotation length. These results are then displayed by the subroutine EVAL (figure 24e).

Output Description and Applications

The complete output of the model FOREST first displays the data pertaining to the plantation being simulated (figure 24a). Then the user has the option of displaying the initial configuration of the stand and the periodic alterations through mortality and thinnings (figures 24b–24d). Finally, the PNW of the plantation in relation to alternative reforestation costs is printed (figure 24e). In a single run the simulation of the growth and management of stands at different initial densities, on alternative sites, under alternative assumptions of costs and benefits and under alternative constraints, can be performed. In that way production functions of the type pictured by figure 5 are easily obtained. If desired these production functions are punched on cards to be used as input in the second part of the Reforestation Planning Model, the program REGEN II. We can then read from the production functions the PNW of a reforestation alternative if we know how many trees per acre this alternative will yield at the initial age,²⁰ if we know the cost of the alternative.

Although the model FOREST has been designed to generate production functions for the Reforestation Planning Model it has a variety of applications as a guide to forest management

²⁰ In all applications an initial age of 10 years has been used. There are two reasons for this choice. First, competition does not alter significantly the diameter growth of Douglas-fir trees before that age (Newnham, 1964). Second, only at about that age can we begin to use diameter breast height as a measure of diameter.

decisions for managed Douglas-fir stands. For example, precommercial thinning decisions can be easily made on the basis of information displayed by figure 5. Given the number of trees that a commercial thinning operation withdraws from a stand and the cost of

the operation we can calculate the PNW added by the operation. Repeating the process for alternative operations we can then determine the best alternative. A wide variety of goals can be encompassed by varying the constraints and the prices.

A MODEL FOR PROJECT ANALYSIS AND RESOURCE ALLOCATION IN REFORESTATION PROGRAMMING

The Model's Function and Structure

The problem at hand consists of finding the set of alternative reforestation activities which yields the plan of maximum present net worth in a given year. This is approached below as follows: first, we define analytically the complete set of available alternatives; second, the reforestation system is represented mathematically so that the *interaction* of all alternatives in their use of common resources is formalized.

Definition of reforestation alternatives. When a tract of land is reforested the following alternatives are available. First, the tract can be partially or wholly reforested. Second, the tract can be seeded or planted. Third, before seeding or planting the site can be prepared in different ways (chemical preparation, scarification, etc.). Fourth, the planting or seed can be carried at alternative densities. Fifth, the tract can be reforested with seeds or seedlings originating from different zone-elevation classes. The Roseburg district, for example, is divided into five seed zones. Each zone is further divided into elevation classes. Regulations currently dictate the way seeds and seedlings can be moved to zone-elevation classes which differ from the ones in which they originate. These rules we take as constraints in the present model.

To quantify all the possible alternatives two types of variables or activities are used in RPM. The primary activities

represent the acreage actually reforested on a given tract according to a particular technique. The secondary activities show the different zone-elevation classes to which the available stocks of seeds or seedlings are allocated, and quantify this allocation.

Evaluation of the alternatives. One assumption is: that given the rules for allocating seeds and seedlings there is no difference in the growth of the trees even though seeds or seedlings originate from different zones. Furthermore, since all the seeds and seedlings are stocked at the district headquarters before being allocated to the different projects, the costs of allocation are the same whatever the origin of the seeds or seedlings used in a particular project. Thus the PNW attached to a secondary activity is zero. Then the primary activities are the only ones which make an explicit contribution to the objective. The evaluation of this contribution in terms of PNW is based upon the production functions generated by the model FOREST. Recall that those production functions show the PNW of one artificially reforested acre of land of specified site quality given the number of trees at age 10 years and given the cost of obtaining that number of trees (figure 5). To determine the PNW of one unit of primary activity we then need the two following functions. First, we need a cost function to calculate the cost of reforesting a specific tract as a function of the technique used. Second, we need a function which forecasts the

number of trees that the considered technique is expected to yield 10 years after application.

Cost functions. When a tract of land is reforested two types of costs are involved. First are the costs independent of the density of the planting or seeding operation. These fixed (on a one acre basis) costs include the cost of site preparation and administrative costs attached to the project. It is assumed that these costs are directly proportional to the size of the reforestation program. Second are the costs which vary with the density of the seeding or planting operation.

Data from McClay (1955) show that, other things equal, the cost of planting, per acre, is directly proportional to the number of trees. On the other hand, Teeguarden (1969) presents a schedule showing the inverse relationship of the cost of planting 1,000 trees to the plantable area. The combination of these two relationships, adjusted for the current costs levels produces the variable cost function used in the Reforestation Planning Model (figure 25).

What the user of the model supplies is a function stating the relationship of the cost of planting 1,000 trees to the plantable area. Then the cost of planting one full acre is calculated as:

$$C = F + C(P)T$$

where

C is the total cost of planting one acre with T trees

F is the fixed cost per acre (independent of density and user supplied)

$C(P)$ is the cost of planting 1,000 trees on a tract of plantable area P .

The type of seeding considered in the

model is aerial seeding from a helicopter. No project characteristic is likely to influence the variable cost of seeding (Teeguarden, 1968). This cost is thus assumed to be directly proportional to the quantity of seeds spread per acre.

Trees survival functions. The only easily available data on reforestation success in the Roseburg district are related to projects executed during the years 1965 and 1966. During that period 321 tracts of land were reforested with Douglas-fir. The following information was reported for each tract:

- the type of treatment (PS), planting or seeding^a
- the size of the tract in acres (AR)
- the location of the tract in the Roseburg district (MU), whether in Drain, North Umpqua, Dillard, or South Umpqua
- the slope of the tract (SL), gentle, medium or steep
- the type of cover (BR and GR), brush or grass: none, light, medium or heavy
- the point-occupancy^b measured at the time or in the first year and third year surveys (PO1 and PO3)

Two statistical regression models were used in an attempt to quantify the effect of variations in a particular site characteristic upon the expected point-occupancy, when all other factors are maintained constant. The first model forecasts directly the point-occupancy three years after treatment as a function of the treatment applied (seeding or planting) and of the site characteristics. The second model forecasts the point-occupancy after three years as a function of the point-occupancy after one year and the site characteristics. The two models use a system of dummy

^a During that period the standard planting and seeding densities were of 680 trees per acre and .75 pounds of seeds per acre.

^b The point-occupancy is a measure of the density of the plantation. It is based upon a sample of 64-square-foot plots systematically distributed on the tract. The point-occupancy is the percentage of plots in the sample which contain one seedling.

TABLE 2
MODELS FOR PREDICTING THE POINT-OCCUPANCY OF A TRACT AT AGE THREE

Variable	Model 1			Model 2		
	Value	S.E.	T	Value	S.E.	T
Point-occupancy after 1 yr. (PO1)...				0.63	0.06	11.20
Area (AR).....	-0.11	0.05	-2.00	-0.07	0.04	-1.60
Treatment (PS) Planting.....	9.00	3.00	3.00	3.00	2.30	1.30
Seeding.....	0.00			0.00		
Slope (SL) Steep.....	-31.00	16.00	-2.00	-17.00	13.60	-1.20
Moderate or Gentle.....	-34.00	16.00	-2.00	-20.00	13.00	-1.40
Flat.....	0.00			0.00		
Aspect (AS) North.....	32.00	16.00	2.00	13.00	13.00	1.60
East.....	32.00	16.00	2.00	18.00	13.00	1.30
West.....	30.00	16.00	1.80	17.00	13.00	1.20
South.....	15.00	16.00	1.50	11.00	13.00	0.80
Flat.....	0.00			0.00		
Grass (GR) Light.....	-13.00	8.00	-1.60	-12.00	7.00	-1.80
Medium.....	-16.00	7.00	-2.30	-9.00	6.00	-1.50
Heavy.....	-23.00	7.00	-3.40	-16.00	6.00	-2.80
None.....	0.00			0.00		
Brush (BR) Light.....	-5.00	7.00	-0.70	-5.00	6.00	-0.70
Medium.....	-10.00	7.00	-1.40	-9.00	6.00	-1.50
Heavy.....	-8.00	7.00	-1.10	-4.00	6.00	-0.70
None.....	0.00			0.00		
Master Unit (MU) North Umpqua.....	5.00	4.00	1.20	1.00	3.20	0.30
South Umpqua.....	-9.00	5.00	-1.70	-3.00	4.10	-0.80
Drain.....	8.00	4.00	2.00	-2.00	3.10	-0.52
Dillard.....	0.00			0.00		
Constants	50.00		6.40	21.00	6.60	3.00
Explained Variance (R ²)	0.21			0.45		

variables which allows for interaction effects.

The forms of the models are:

Model 1: $PO3 = -.11$
 $AR+PS+SL+AS+GR+BR+MU+C1$
Model 2: $PO3 = .63PO1 - .07$
 $AR+PS+SL+AS+GR+BR+MU+C2$

where C1 and C2 are constants. The expected values of all the variables are given in table 2. For example, suppose

we seed a 10-acre tract on a moderate slope, north aspect,²³ with no grass, medium brush and located in Dillard, with .75 pounds of seeds. Then Model 1 would forecast an expected point-occupancy after three years:

$PO3 = -1.1 + 0. -34. +32. +0. +10. +0. +50.$
 $PO3 = 36.9$ per cent.

Unfortunately the coefficients of determination of the two models are very

²³Qualification of the models is necessary before they are used. Seeding is usually carried on in conditions which are relatively good for reforestation. Thus the models should not be used to compare the results of seeding in conditions where seeding is not usually performed.

small ($R^2 = .21$ and $.45$ respectively), which confirms the fact that many other factors in addition to site characteristics influence the success of a reforestation project. However, several variables are significant at the 95 per cent confidence level. Hypothesis testing gave the following results, everything else equal:

1—Point-occupancy decreases with tract size.

2—Planting yields a higher point-occupancy than seeding.

3—The variation in point-occupancy between north slopes and south slopes is not significant.

4—Point-occupancy is lower on tracts having some slope, but the difference between steep slopes and moderate or gentle slopes is not significant.

5—Point-occupancy decreases with the presence of grass cover, but the difference between light and heavy grass is not significant.

6—Point-occupancy is not significantly influenced by brush cover.

7—Point-occupancy in 1965 was the same as in 1966.

8—Point-occupancy is significantly higher on tracts in Drain, lower on tracts in South Umpqua, average on tracts in North Umpqua.

Despite the fact that some of the variables in the model are not statistically significant at the 95 per cent confidence level, most of them were retained in the forecasting model.²⁴

Once the forecast of the point-occupancy three years after the reforestation is determined we calculate the expected number of trees using the relation:

$$\begin{aligned} \text{Number of trees} = \\ \text{Point-occupancy} \times 680. \end{aligned}$$

It is assumed that this is the number of crop trees per acre present on the

tract at age 10. This number, which results from the planting of 680 trees or from the seeding of 0.75 pounds of seeds, is called the expected *standard stocking* of the tract.

Given the lack of precision of the models presented above, the RPM allows the user to replace this method of forecasting by any available means. The result can be entered as part of the data set for the model REGEN II.

Once the forecast of the expected standard stocking is obtained, the expected stocking at age 10 is calculated as follows:

$$E_{10} = \frac{ES \times D}{S}$$

where

E_{10} is the expected stocking at age 10 (trees per acre)

ES is the expected standard stocking at age 10 (number of trees per acre resulting from planting 680 seedlings or seeding .75 pounds per acre)

D is the actual density applied (number of seedlings planted or pounds of seeds seeded per acre)

S is the standard density (680 seedlings or .75 pounds of seeds per acre).

Value added computation. Once we have the expected stocking at age 10 and the cost attached to a particular alternative, we can determine its value. In order to keep the number of alternatives in the RPM to a minimum we take all the "do nothing" alternatives as bases of comparison and calculate the PNW added by all other alternatives above what doing nothing would yield. The "do nothing" alternative is what would be obtained at zero cost from natural regeneration. For example, if natural regeneration has a PNW of 50 dollars per acre on a given tract while planting 300 trees has a PNW of 150 dollars,

²⁴ This is justified for three reasons. First, omission of a variable which should be in the model leads to biased values of the coefficients in the model. Second, most of the coefficients obtained have the relative values that prior knowledge would lead us to expect. Finally, it is hoped that the precision with which the coefficients are determined can be improved by future studies.

then the PNW added by planting 300 trees is 100 dollars.

Linear programming model. The model used to represent the interaction of all the projects in the reforestation system belongs to the well-known class of linear programs. In such a model the objective function as well as the constraints are linear functions of the activities.

As noted earlier, the objective of the manager of a reforestation program is to maximize the present net worth of the total program. This we write:

$$\max \sum_{t=1}^N \sum_{a=1}^{a_t} X_{ta} P_{ta}$$

where

X_{ta} (primary activity) is the acreage of tract t reforested according to the treatment alternative a .

P_{ta} is the PNW added by applying alternative a to tract t .

N is the total number of tracts available for reforestation.

a_t is the number of treatment alternatives which can be applied to tract t .

Three types of constraints limit the size of the reforestation program. First, is the quantity of land available. Second, is the size of the budget. Third, is the limitation of the stocks of seedlings and seeds.

The *budget constraint* expresses the fact that the funds used by the program cannot exceed the amount available. Thus we have:

$$\sum_{t=1}^N \sum_{a=1}^{a_t} X_{ta} b_{ta} \leq B$$

where

b_{ta} is the amount of money necessary to reforest one acre of tract t according to the treatment alternative a .

B is the total amount of money available.

The fact that the quantity of seedlings (seeds) used cannot exceed the available stocks can be represented by a system of constraints which uses the primary activities and the secondary activities defined above. Thus the system of constraints is expressed by:

$$\text{for all } c: \sum_{a=1}^{a_t} X_{ta} d_a - \sum_{s \in C} K_{cs} = 0.$$

$$\text{for all } s: \sum_{c \in S} K_{cs} \leq K_s$$

where

c is a zone-elevation class in which there are tracts to reforest.

s is a zone-elevation class where part of the stock of seedlings (seed) originate.

d_a is the density of seedlings (seeds) used by treatment alternative a .

K_{cs} is the amount of seedlings (seeds) from zone-elevation s allocated to zone-elevation c .

C is the set of zone-elevation classes which can supply zone-elevation c .

S is the set of zone-elevation classes to which seedlings (seeds) from zone-elevation class s can be allocated.

K_s is the available stock of seedlings (seeds) originating from zone-elevation s .

A final set of constraints expresses the fact that the acreage reforested on a particular tract can at most be equal to the area of that tract.

$$\text{for all } t: \sum_{a=1}^{a_t} X_{ta} \leq A_t$$

where A_t is the area of tract t .

This system of linear equations completely defines the resource allocation sub-model of RPM. For practical application, a model is needed to analyze rap-

idly the extraordinarily large number of feasible plans and to select the optimum plan. Such a model is presented below.

The Model REGEN II

The model described in this section is composed of two computer programs REGEN II and FINAL. The program FINAL is only an editing device which clarifies the results of REGEN II (figure 26). The model REGEN II has four purposes. First, the subroutine ALTERN computes the PNW of alternative treatments applied to a tract of land. Second, the subroutines SUPPLY, SELECT, ROWS, COLUMN, and RHS construct the linear programming model which describes the system of interdependent activities, constraints, and objectives. Third, the subroutine PRELIM calculates a preliminary solution to the allocation problem. This preliminary solution is obtained by ranking all the available tract-treatment alternative combinations according to one of three criteria, subject to the resource constraints. The criterion can be the PNW per acre, the PNW per dollar invested, or the PNW per seedling (pound of seed). The rationale supporting these criteria was discussed earlier. If a single resource (e.g., budget) is constraining, PRELIM yields the optimum plan. Otherwise only an initial feasible solution of the linear program is obtained. This solution is then added to the data generated by the subroutines ROWS, COLUMN, RHS to be used as input to the subroutine MPS. This subroutine then derives the optimum solution of the linear program through an iterative

process (simplex) whose starting point is the solution calculated by PRELIM. The optimum solution is then produced on punched cards used later by the program FINAL to display the reforestation plan. The remainder of this section describes in more detail the function of each of the elements of the model REGEN II.

Project Analysis. The project analysis performed by REGEN II uses the production functions prepared by the model FOREST. These functions are read into the system and displayed (figure 27a) by the subroutine READ. This same subroutine also reads data concerning all the treatment alternatives available to execute the total reforestation program (figure 27b). These data include several treatment packages defining a plant or seeding operation with a particular site preparation and a set of alternative densities. The fixed cost of each treatment package is given in the data set. Also given are the variable costs of planting 1,000 trees (as a function of the plantable area), and the variable cost of seeding one pound of seeds.

The project analysis proper is executed by the subroutine ALTERN (figure 28a). Each project is completely defined by the information contained in a single data card. This information includes the tract reference (three alpha-numeric characters), the zone-elevation class, the master unit, the site index, the aspect, the grass and brush cover, the slope, the area plantable, the expected number of trees at age 10 if no treatment is applied,²⁵ the reference of the planting and/or seeding treatment packages to be used on the tract,²⁶

²⁵ In all the applications of the model to the Roseburg district this number was set to 0 for bare land (thus neglecting natural regeneration) and to the number of trees revealed by the surveys in the case of inter-planting. However the manager who can estimate the number of crop trees at age 10 which would result from natural regeneration should use this knowledge to determine the expected number of trees at age 10 if no treatment is applied.

²⁶ A single planting package and a single seeding package can be specified on a card. Thus, if alternative treatment packages are to be compared, the same tract must be filed several times in the data set.

and, finally, the expected standard stocking at age 10. If the expected standard stocking is not specified by the user it is calculated by the function subprogram POCC using the regression Model 1 of table 2.

Then the subroutine ALTERN proceeds to calculate the PNW of the tract if no treatment is applied, and the added PNW corresponding to each alternative treatment. Each PNW is extracted from the production functions by the subroutine EXPVAL using a double interpolation procedure.

The subroutine ALTERN finally displays the results of the project analysis for each tract and for each treatment package (figure 28b).

Supply of resources and activities screening. If a solution to the allocation problem is desired after execution of the project's analysis, a set of data defining the available resources has to be supplied. This data set contains the available budget as well as the stocks of seeds and seedlings. The data set is read and displayed (figure 29) by the subroutine SUPPLY. The data definition allows the user to indicate a preferential order regarding the zone-elevation classes to which a given stock of seedlings or seeds can be allocated.

Another function of the subroutine SUPPLY is to eliminate the treatment alternatives rendered infeasible by the supply constraints. During the procedure each tract which is scheduled for planting (seeding) but which cannot be supplied is listed by the subroutine SUPPLY.

After this first screening the subroutine SELECT is used to select the relevant activities attached to each tract. SELECT eliminates first the activities with a PNW = 0. Then it discards all the planting or seeding densities which do not yield an increase in PNW.

Contruction of the linear program. At the core of the model REGEN II is

the linear program whose mathematical structure was described above. To be solved by the subroutine MPS, this linear program has to be presented according to a special format (I.B.M., 1968). The resulting data matrix (figure 30) is composed of three parts. The ROWS part contains the names of all the equations, inequations and objective function of the model. The COLUMNS part defines all the column vectors. The RHS part defines the right-hand side of the program. These three parts are generated by the subroutines ROWS, COLUMN, RHS of the model REGEN II and recorded on a disc where they are retrieved by the subroutine MPS.

The subroutine ROWS (figure 31) names first the objective function and the budget constraint of the model. Then, one consumption constraint is named for each zone-elevation class which contains at least one tract with a relevant reforestation alternative.⁷ The same subroutine also names one supply constraint for each zone-elevation class where stocks originate and which can supply a zone where stock is needed. Finally ROWS names an area constraint for each tract with some relevant reforestation alternative.

The subroutine COLUMN (figure 32) generates and records the column vectors of the linear programming model. Two types of vectors are involved corresponding to the primary secondary activities of the model. There is one primary activity for each relevant reforestation alternative. The primary activities are always expressed in total acres so the input-output coefficients corresponding to one acre of activity are reduced by the plantable area coefficient k . Thus the PNW per acre of activity on a tract which is k -tenth plantable becomes $PNW \times k/10$. The planting cost on the tract is reduced by the same coefficient, as is the density of planted seedlings per acre. However, the density

⁷ By relevant reforestation alternative is meant a seeding or planting operation which can be supplied with stock and which has a positive PNW.

of seeds per acre and the cost of seeding remain the same since the area plantable does not alter those factors.

After all the column vectors corresponding to the primary activities have been generated and recorded the subroutine COLUMN proceeds to generate and record the column vectors corresponding to the secondary activities. Such a column vector is produced for each combination of zone-elevation classes requiring stock with zone-elevation classes which can supply the stock.

The subroutine RHS (figure 33) generates and records the right-hand side of the linear programming model. First, the budget is recorded as the right-hand side of the budget constraint. Second, the available stocks which can be allocated to some projects are recorded as the right-hand sides of the corresponding supply constraints. Third, the area of each tract with some relevant reforestation activity is recorded as the right-hand side of the corresponding area constraints.

Solution of the allocation problem.

Recall that the solution of the allocation problem is determined in two steps. First a preliminary solution is determined by the subroutine PRELIM. Second, the optimum solution is calculated by the subroutine MPS and displayed by the program FINAL.

The first function of the subroutine PRELIM (figure 34) is to evaluate each of the treatment alternatives (reforesting a particular tract by planting or seeding with a certain site preparation and a particular density) using one of the three following criteria:

- present net worth per acre
- present net worth per dollar invested
- present net worth per seedling (or pound of seeds) used.

When each alternative has been evaluated the subroutine PRELIM ranks

all alternatives according to the designated criterion. Then, the subroutine proceeds to select a set of projects composed of the alternatives which perform best according to the criterion while conforming to the following constraints. First, there must be a single alternative per tract. Second, the selected set cannot use more funds than are available. Third, the set of alternatives can use at most the available stock of seedlings and seeds, while respecting the prevailing rules of allocation. Within these rules of allocation, the subroutine PRELIM attempts to observe the preferences indicated in the data relating to the availability and potential allocation of the stocks.

When a tract is selected by the subroutine PRELIM, the following information is displayed (figure 35): first, the tract's code, zone-elevation class and effective area;²⁰ then, the cost of the treatment alternative selected and its added PNW; finally, the amount of planting (or seed) stock required as well as the zone-elevations from which the stock originates is presented. After the set of all the tracts corresponding to the initial solution has been displayed the subroutine PRELIM produces the total area treated, the total PSV added, and a full description of the remaining stocks of money, seedlings and seeds.

If a solution to the linear program is required by the user, the solution calculated by PRELIM is stored on a disc where it is retrieved by the subroutine MPS to serve as the initial solution of the linear program. This stored solution is composed of a set of primary activities and secondary activities describing the solution selected by PRELIM.

The solution of the linear program is obtained by the subroutine MPS.²¹ The functions performed by the subroutine are guided by a control program which can be found in the program listings.

²⁰ The effective area is the full area of the tract multiplied by the plantable area ratio.

²¹ The Mathematical Programming System is a computer program designed by the International Business Machines Corporation to solve linear programming problems (I.B.M., 1968).

The first step of MPS is to transform the initial solution produced by PRELIM into a basic feasible solution with unchanged PNW added. Then MPS proceeds to improve this solution by successive iterations using the revised simplex algorithm.³⁰

After obtaining the optimum solution the subroutine MPS produces this solution on punched cards. These cards are then used as single input into the last program FINAL which displays the reforestation plan (figure 36). The program FINAL groups together all the tracts in the same zone-elevation class and shows the amount and origin of the stocks to be used on those tracts. For each tract the area treated according to each alternative treatment and density is reported by the program.³¹

Application. The model described in the preceding sections has been designed primarily for short-range planning. In a typical application, the user would first run the part relating to project analysis. He would then check the results and perhaps revise some of the predicted standard stockings. On a second run the user could attempt to obtain a preliminary solution using the ranking criterion which seems most proper to the situation at hand. Observing which tracts are rejected because of

the lack of resources, he could then revise the rules of allocation of the seedlings and seeds. If more than one stock of resources is exhausted by the preliminary solution and if a considerable amount of land is left untreated, it is likely that the preliminary solution can be improved. Then, the model could be run a last time to calculate a solution using the linear programming model.

Since many characteristics of the reforestation system can change during the execution of the projects, the plan may have to be revised. The model can then be used repeatedly to determine the new plan as a function of the remaining stock of resources.

The model can also be used for long-range planning to determine the quantity of resources necessary several years hence. In that case, the most useful element of the model would be the part relating to project analysis. Given the future conditions of the tract to reforest and the costs of reforestation, one could determine the best treatment for each tract and use that information to plan the future budget and the production of seedlings. In that way, we obtain the link between the reforestation subsystem modelled here and the subsystems which constitute its environment.

THE REFORESTATION PLANNING MODEL: AN ILLUSTRATIVE APPLICATION

In this chapter, the results of a practical application of RPM are described. The planning unit considered is the Roseburg district in Southwestern Oregon, in the year 1970.

The Reforestation Planning Problem

Available projects and treatments.

In the year 1970, the Roseburg district had 312 tracts of land to consider for

³⁰ A description of the revised simplex algorithm can be found in Dantzig (1963) under the title: "Simplex method using multipliers."

³¹ Given the nature of the linear programming solution, the plan sometimes allows for different treatments (for example, two different densities, or planting and seeding) to appear on the same tract. The user must then select a practical solution so that each tract in the plan is scheduled for one treatment.

reforestation. The tracts ranged in size from less than 10 acres to more than 100 acres. The total area available was 9,027 acres. Four site qualities were represented (sites II, III, IV and V). Among the 312 tracts, 103 were totally bare of trees while others were stocked with densities ranging from less than 50 to more than 200 trees per acre.

In 1970, aerial seeding was precluded by existing policy. Aerial seeding requires the coating of the Douglas-fir seeds with enderine, a chemical compound currently banned by law from all federal lands. Some tracts had been invaded with grass and it was decided that if those tracts were to be reforested at all they would require a chemical grass control treatment before planting. Thus, two alternatives were available, planting alone and planting with chemical grass control. These two alternatives were mutually exclusive. Furthermore each alternative could be executed using five alternative densities: 50, 150, 300, 680 or 1,210 trees per acre.

The cost of the treatments was estimated on the basis of experience from the previous year. Fixed costs were \$2.50 per acre for planting alone, and \$16.50 for planting with chemical grass control. Variable costs were estimated using as a basis the cost of planting 1,000 trees as a function of the plantable area.

Plantable Area	Planting Cost
Per cent	\$ per M Trees
10	\$112
20	112
30	112
40	112
50	67
60	59
70	56
80	55
90	55
100	55

Supply of seedlings and budget. A

total of 1,259,000 seedlings were available. The seedlings originated from 10 different zone-elevation classes and could be distributed to 28 alternative zone-elevation classes according to the following rules. First, seedlings could be transferred between seed zones as shown in figure 37. Second, seedlings originating from a particular elevation could be allocated 500 feet lower.

Determination of the limiting budget was not an easy task. Since reforestation is a high-priority investment in forest management, the necessary money has always been easy to obtain. On the basis of expenditures in past years an arbitrary sum of 150,000 dollars was used as a provisional upper bound to the amount of money which could be spent. It was expected that a considerable percentage of this amount would be left unspent.

Generating the Value Production Functions with the Model FOREST

In this application we used a rate of interest of three per cent per year and a price of timber of 70 dollars per thousand board feet for timber having an average diameter breast height of 20 inches.

The derivation of these data was somewhat arbitrary. A rate of three per cent has been commonly used by federal agencies to evaluate public investments (Eckstein, 1958; U.S.D.A., 1963). This figure is also used by conservative private firms of the Douglas-fir region in the long-range planning of the management of their forest properties (U.S. D.A., 1963). Furthermore, a recent study by the federal reserve bank of St. Louis (Yohe and Karnosky, 1969) shows that the real interest rate on long-term grade A bonds has remained around three per cent during the last ten years (figure 38).

Seventy dollars per thousand board feet is a forecast of the price forty years from now; forty years is the time it

takes currently established plantations to develop to full production. The projection was made using a sigmoid curve (figure 39) to allow for some decreasing rate in the trend.

In the present example, the purpose is not to determine the amount of resources which should be allocated to reforestation. This amount is taken for granted, because it is fixed by the size of the budget. The purpose of prices in RPM is to furnish a common basis upon which all the available projects will be compared. As stated earlier, we do not want a set of prices that will result in the *a priori* rejection of a project. The interest rate of three per cent and a price of 70 dollars per thousand board feet yield a positive PNW even on a project which requires 200 dollars per acre to obtain 50 trees on site V land. In this way, no project in the Roseburg district is discarded at the project analysis stage. Thus, the limitation of resources in the district is the only factor limiting the number of projects which are executed. Timber prices are used simply to derive the relative value of the projects so that only the best ones are executed.

It was assumed in the derivation of yield estimates that the minimum rotation length acceptable would be 50 years, whatever the economic optimum indicated by the model. The price difference between final harvest outputs and thinnings was set at five cents per cubic foot. The minimum volume acceptable to justify a thinning operation was 500 cubic feet. The minimum merchantable size of timber was seven inches in diameter at breast height. Finally, the initial stand characteristics were assumed to relate to the site index of the land in the following way:

Characteristic	Site			
	II	III	IV	V
Average diameter age 10 (inches)	2.2	1.8	1.0	0.5
Standard deviation (inches)	1.1	.9	.5	.2

Under the assumptions noted the production functions in figures 40a-40d were developed. The isocost curves show a maximum value per acre for a density somewhere between 680 and 1,210 trees per acre at age 10. On site II, however, a maximum was not yet reached at a density of 1,210 trees. The PNW of projects ranged from a low of 100 dollars per acre for site V land requiring 90 dollars per acre to yield 50 crop trees at age 10 to a high of 191 dollars per acre for site II land yielding 1,210 trees per acre with no expenditure.

Computing the Optimal Reforestation Program

All the production functions produced by FOREST were loaded as data into the program REGEN II. Added were data defining the alternative treatments and the resources limitations. The data relevant to each one of the tracts were also supplied.

The forecast of the standard number of trees was entirely left in this example to the internal function of the model. All projects were found to have at least one alternative with a positive PNW. Given the accounting price of seedlings used before taking into account all the constraints, the best planting density came out to be 680 trees per acre for 40 tracts of land and 1,210 trees per acre for 272 tracts. Execution of all the projects with the optimum density of trees would have yielded a plan with a PNW of 5,641,724 dollars, had all the necessary seedlings been available at the current prices.

It was apparent at the outset that there were insufficient seedlings to permit execution of all projects using the best density. Thus, the criterion used to generate a preliminary solution to the allocation problem was the ratio of the PNW per acre to the density of seedlings used.

The preliminary plan, which selects the alternatives performing best accord-

ing to the criterion while respecting the resources constraints, gave the following results. The PNW of the plan was 2,018,218 dollars. The unused budget was 71,459 dollars and 363,216 seedlings were left unused. A total number of 265 tracts of land were scheduled for reforestation, totalling 7,391, acres. The densities prescribed varied from 50 to 300 trees per acre.

The linear programming solution led to a plan with 228 tracts totalling 6,355 acres of land scheduled for reforestation. The PNW of the plan was 2,558,554 dollars, 25 per cent higher than the PNW yielded by the preliminary solution. No seedlings were left in the stocks while 61,064 dollars were left unspent. The prescribed densities varied from 50 trees per acre to 680 trees per acre, in the following proportions:

Densities	50	170	300	680	Total
Acres	1824	2632	2272	90	6355

Conclusions

This example clearly depicts the trade-off which exists between the total area reforested and the density at which reforestation is executed. The final solution achieves a higher PNW by treating less than the preliminary plan but by

using higher densities and all the available stocks.

Most of the time the linear programming solution had an integer form. That is to say, a single alternative is generally assigned to a particular tract, despite the fact that no constraint appears in the model for this requirement to be satisfied. Only four tracts were scheduled to be partially reforested with two different densities. There would be no problem for the manager to define a single density which when used on the total tract would make the same requirement in seedlings.

The linear programming solution has a PNW substantially higher than the simple ranking solution. The shorter the supply of seedlings and seeds, the more useful the linear programming model. The linear programming solution requires 20 minutes of computer time on the I.B.M. system 360/40. Thus, the expense (about 100 dollars) is fully justified by the extra benefits yielded by the solution. Generating the production functions of the example requires 15 more minutes of computer time. But, a new set of production functions is needed only when the assumptions stated above cease to hold.

SUMMARY AND CONCLUSIONS

The purpose of this concluding segment is to summarize briefly the results of the study, to suggest some improvements and extensions of the Reforestation Planning Model, and to report briefly the response that the model has received from Bureau of Land Management users.

Principal Results

We have attempted to design a model to assist managers in planning public reforestation programs in the Douglas-fir region. The two principal planning problems are: which projects should be

executed at a certain point in time, and what method of reforestation should be applied within a given set of resource constraints?

The fundamental rule which governs the allocation of resources and the evaluation of projects is the maximization of present net worth. This rule appears to be the most consistent with the directives of the federal government regarding the management of commercial timber land. However, the market should not be the sole basis for the determination of the timber prices used in the calculation of present net worth. In ad-

dition other factors which are not evaluated by the market should be taken into account in evaluating projects, and as a consequence the accounting prices should be modified.

The final outcome of the study is a set of computer programs with the following functions:

One program, **FOREST**, simulates a regime of harvests on plantations consistent with the goal of PNW maximization. It also generates a set of production functions which relate the PNW per acre of a plantation to the number of trees age 10 and to the cost of obtaining that number of trees.

The second program, **REGEN II**, first calculates the value per acre of different alternatives which can be applied in reforesting a particular tract of land. Then, the same program determines the set of alternatives which maximizes the PNW of the reforestation program while respecting the resources constraints.

Reading the final output given by the program **FINAL**, the manager finds a guide indicating which tracts to reforest and which method to use.

Possible Improvements and Extensions

The growth function which predicts the diameter growth of a Douglas-fir tree was derived indirectly from aggregate growth functions. It would be interesting to analyze quantitatively the factors which influence the growth of a tree in a managed stand and to insert the resulting function into the model. **FOREST** would readily accept a function with such independent variables as the current diameter of the tree, its age, its crown class, the density around the tree in basal area, and the diameter of the trees constituting this density.

In the current version of the model, one function specifically relates to the Roseburg district. This is the function forecasting the standard number of

trees per acre resulting from a planting or a seeding operation. To use the model in other parts of the Douglas-fir region one would have to insert a function relevant to the area of application, or one could use the model as such but specify the standard number of trees for each project in order to bypass the forecasting equation. As we have seen earlier, the current equation in the model is not precise. An immediate improvement of the model could be realized by carefully studying the relation of the expected stocking in the reforested areas to all the factors surrounding the planting or seeding operations.

One serious limitation of the model **REGEN II** is its static character. Contrary to what the model assumes, some resources unused in one period (e.g., money and seeds) do not have zero value since they can be stored for later use. There is no reason why the model could not be expanded to include more than one time period. However, this extension would rapidly increase the size of the model to render it unsolvable in a reasonable amount of computing time. One way of limiting the number of activities and of constraints would be to group projects into classes of similar characteristics instead of considering each project individually.

Test Applications

The application of the model to real planning situations seems promising. In test applications, the Bureau of Land Management praised the model's built-in flexibility which permits it to adapt the design of the reforestation plan to certain management constraints and policies. The rotation age on forest lands managed by the Bureau is an example of such a constraint. Also valued is the attempt to align the model as close to the conceptual framework familiar to the field foresters in charge of reforestation as possible. At least one design feature has been chosen so as to

involve these foresters in the planning procedure in a more challenging way than merely the collection of data. We refer here to the prediction of the standard stockings in which field experience should make the most valuable contribution.

Managers interested in areas of planning other than reforestation have shown considerable interest in the model FOREST. They seem ready to experiment with the model in precommercial thinning decisions and other fields where production functions for man-

aged-even-aged-Douglas-fir stands are necessary.

However, only extensive experience with the model will reveal its usefulness and its deficiencies. Furthermore, a computer system can become rapidly obsolete, unless constantly revised. It is the hope of the authors that this document will help future users to better understand what assumptions the model is based on, how these assumptions are built into the model, and how they can be modified if they become too far removed from reality.

ACKNOWLEDGMENTS

This study was conducted in the California Agricultural Experiment Station in cooperation with the U. S. Department of Interior, Bureau of Land Management, which provided financial support, data, facilities, and assistance from personnel in State and District Offices in Portland and Roseburg, Oregon, respectively. The authors are greatly indebted to the following persons who provided help during the various stages of the study: Murl Storms, Chief of the Division of Re-

source Management, Washington, D.C.; Herbert Hagland, Chief of the Division of Resource Management, Portland, Oregon; George Francis, District Manager, Roseburg, Oregon; Bernard Mayer, Resource Manager, Roseburg; Ronald Kaufman and David Nelson, Economists, Service Center, Portland.

The authors also wish to express appreciation for the efforts of Professor Rudolf F. Grah and Dr. James A. Baker, who reviewed the manuscript and provided helpful comments.

LITERATURE CITED

Published Materials

AMIDON, E. L. and G. S. AKIN

1968. Dynamic programming to determine optimum levels of growing stock. *Forest Science* 14(3):287-291.

ASSMAN, ERNST

1953. Density of stocking and wood production. *Management* 26 Trans. from Forstmiss Centrbl. 72(3/4): 69-101. U. S. Forest Service, Washington, D.C.

BAUMOL, WILLIAM J.

1965. *Economic theory and operations analysis*. Prentice Hall, New Jersey.

CHAPPELLE, DANIEL E. and THOMAS C. NELSON

1964. Estimation of optimum stocking levels and rotation ages of Loblolly Pine. *Forest Science* 10(4):471-502.

CHISLOM, ANTHONY H.

1965. Criteria for determining the optimum replacement pattern. *J. Farm Econ.* 48(1):107-112.

CURTIS, ROBERT O.

1967. A method of estimation of gross yield of Douglas-fir. *Forest Science Monograph* 13. Soc. of Amer. Foresters, Washington D.C.

DANTZIG, GEORGE B.

1963. *Linear programming and extensions*. Princeton Univ. Press, Princeton, New Jersey.

- DUERR, WILLIAM A.
1960. Fundamentals of forestry economics. McGraw Hill, New York.
- DUFF, G.
1956. Yield of unthinned Douglas-fir, Corsicom pine, and Ponderosa pine in New Zealand. New Zealand Forest Service, Wellington. Forest Res. Note 5.
- ECKSTEIN, OTTO
1958. Water resources development. Harvard Univ. Press.
- FEDKIW, JOHN
1968. Planning forest resource management in U.S.D.A. J. Forestry 66(10):768-773.
- FISHER, IRVING
1954. The theory of interest. Kelly and Millman, New York.
- HIRSHLEIFER, JACK
1958. On the theory of optimal investment decision. J. Political Economy 66:329-352.
- HOYER, GERALD E.
1966. Optimum thinning intensity tables. DNR Report No. 7. Washington Dept. of Natural Resources. Olympia.
- INTERNATIONAL BUSINESS MACHINES
1968. Linear and separable programming user's manual. H20-0476 360/OS MPS/360 (360-CO-14X).
- KER, JOHN W. and J. HARRY G. SMITH
1955. Advantages of the parabolic expression of height-diameter relationship. Forestry Chronicle 31(3):236-246.
- LITTLE, I. M. D. and J. A. MIRRELES
1968. Manual of industrial project analysis in developing countries. Vol. 2, Social Cost-Benefit Analysis O.C.D.E. Paris.
- MCARDLE, RICHARD E., WALTER H. MEYER and DONALD BRUCE
1961. The yield of Douglas-fir in the Pacific Northwest. Washington, D.C. U.S.D.A. Tech. Bull. 201.
- MCCLAY, T. A.
1955. Economic considerations in Douglas-fir stand establishment. U. S. Forest Service. Research Paper 15. Pacific Northwest Forest and Range Experiment Station, Portland, Oregon.
- MÖLLER, CARL M.
1954. The influence of thinning on volume increment. Thinnings Problems and Practices in Denmark. Tech. Bull No. 76. New York State College of Forestry at Syracuse Univ., Syracuse, New York.
- NASLUND, BERTIL
1969. Optimal rotation and thinning. Forest Science 15(4):446-451.
- NEWHAM, R. M.
1964. The development of a stand model for Douglas-fir. Ph.D. Thesis, Faculty of Forestry, Univ. of British Columbia, Vancouver.
- REUKEMA, DONALD L.
1970. Forty-year development of Douglas-fir stands planted at various spacings. U.S. Forest Service Research Paper 100. Pacific Northwest Forest and Range Experiment Station, Portland, Ore.
- SAMUELSON, P. A.
1964. Economics, an introductory analysis. Sixth edition. McGraw Hill, New York.
- SMITH, J. H. G., J. W. KER and J. CZIMZMAZIA
1961. Economics of reforestation of Douglas-fir, Western Hemlock and Western Red Cedar in the Vancouver forest district. Forestry Bulletin No. 3. Univ. of British Columbia, Vancouver. 144 pp.
- SPURR, STEPHEN H.
1962. A measure of point density. Forest Science 8(1):85-96.
- STAEBLER, GEORGE R.
1960. Theoretical derivation of numerical thinning schedules for Douglas-fir. Forest Science 6(2):98-109.
- TEEGUARDEN, DENNIS E.
1969. Economic criteria and calculated risk in reforestation investment decisions. J. Forestry 67(1):25-31.
- TEEGUARDEN, DENNIS E. and H. S. VON SPERBER
1968. Scheduling Douglas-fir reforestation investments: a comparison of methods. Forest Science 14(4):354-367.

U. S. DEPARTMENT OF AGRICULTURE, FOREST SERVICE

1963. Timber trends in Western Oregon and Western Washington. U.S.F.S. Pacific Northwest Forest and Range Experiment Station Research Paper PNW 5. Portland, Oregon.

U. S. DEPARTMENT OF INTERIOR, BUREAU OF LAND MANAGEMENT

1968. Manual 1602, basic guidance. Washington, D.C.

WEINGARTNER, H. M.

1966. Capital budgeting of interrelated projects, survey and synthesis. *Management Science* 12(7):485-516.

YOHE, WILLIAM P. and DENIS S. KARNOSKY

1969. Interest rates and prices level changes, 1952-1969. Federal Reserve Bank of St. Louis, Reprint Series No. 49. 38 pp.

Unpublished Materials

DEBREU, GERARD

1970. Lecture notes from a course given by Gerard Debreu in Linear Programming, Univ. of Calif., Berkeley.

RADNER, ROY

1960. Lecture notes on the Theory of Economic Planning. Athens.

RADNER, ROY

1970. Lecture notes from a course on National Economic Planning. Univ. of Calif., Berkeley.

U. S. DEPARTMENT OF INTERIOR, BUREAU OF LAND MANAGEMENT

1965. A new stand-model approach to determine Douglas-fir rotations in Western Oregon. Unpublished paper. Portland, Oregon.

Fig. 1. Trend of federal reforestation investments in Oregon and Washington.

Fig. 2. Sequence of planning and operations in BLM's Douglas-fir reforestation program in Oregon. (From: Teeguarden and Von Sperber, 1968.)

Fig. 3. The Roseburg district in southwestern Oregon.

Fig. 4. The Reforestation Planning Model.

Fig. 5. Example of the relationship of present net worth to initial stocking and establishment costs. Underlying assumptions are in chapter 5.

Objective function: (to maximize)		Present net worth of the reforestation program		
	Budget	Money used	≤	Money available
	Seedlings Supply (∑i)	Seedlings from zone-elevation i consumed	≤	Seedlings from zone-elevation i in stock
	Seeds Supply (∑i)	Seeds from zone-elevation i consumed	≤	Seeds from zone-elevation i in stock
	Land (∑i)	Area of project j treated	≤	Area of project j available

Fig. 6. Structure of the linear programming model.

Fig. 7. Optimum investment policy.

Fig. 8. Project selection when a single resource is constraining.

Fig. 9. Relationship of internal rate of return criterion to present net worth.

Fig. 10. Stock accumulation and thinning in the forest stand.

Fig. 11. Present net worth of two reforestation alternatives on site index 90.

Fig. 12. Flow chart of the stand growth model.

Fig. 13. Initial stand representation as performed by the program subroutine INITIA.

Fig. 14. Value of timber as function of tree diameter.

Fig. 15. The subroutine EXPAND.

Fig. 16. Search pattern around a central tree C of the working grid (subroutine SEARCH).

Fig. 17. Tree mortality simulation by the subroutine subprogram MORTAL and the function subprogram PDTH.

Fig. 18. Diameter growth function on a given site.

Fig. 19a. Predicted and observed survival in plantations.

Fig. 19b. Predicted and observed diameter growth in plantations.

Fig. 19c. Predicted and observed net basal area in plantations.

Fig. 19d. Predicted and observed net volume in plantations.

Fig. 20. The decision-making system of the model FOREST.

Fig. 21. Tree-harvesting decision as performed by the subroutine THIN.

Fig. 22. Determination of optimum stocking by the thinning algorithm.

- a) site : 110
- b) initial spacing : 8 x 8 ft.
- c) value : \$.0175/cubic feet/inch
- d) interest rate : .03 per year

Fig. 23a. Economic stocking level as a function of age.

Fig. 23b. Present net worth of unthinned and thinned stands.

PER ACRE DATA	, SITE 110.	, IN. TREES 121C.	, MANAGEMENT	, INT. RATE 0.03
D.B.H.	STAND AT AGE 10.			
1	557.			
2	653.			
3	0.			
4	0.			
5	0.			
6	0.			
7	0.			
8	0.			
9	0.			
10	0.			
11	0.			
12	0.			
13	0.			
14	0.			
15	0.			
16	0.			
17	0.			
18	0.			
19	0.			
20	0.			
21	0.			
22	0.			
23	0.			
24	0.			
25	0.			
26	0.			
27	0.			
28	0.			
29	0.			
30	0.			
TOTAL	1210.			
AV. D	1.0			
VOL	0.			
VALUE	0.			
PNV	0.			

Fig. 24b. Stand at initial age displayed by the subroutine INITIA of the program FOREST.

PER ACRE DATA	, SITE 110.	, IN. TREES 1210.	, MANAGEMENT 1	, INT. RATE 0.03
D.B.H. CLASS	THINNING AT AGE 10.	MORTALITY FROM AGE 10. TO 15.	STAND AT AGE 15.	
1	0.	36.	0.	0.
2	0.	12.	0.	0.
3	0.	0.	617.	0.
4	0.	0.	508.	0.
5	0.	0.	0.	0.
6	0.	0.	0.	0.
7	0.	0.	0.	0.
8	0.	0.	0.	0.
9	0.	0.	0.	0.
10	0.	0.	0.	0.
11	0.	0.	0.	0.
12	0.	0.	0.	0.
13	0.	0.	0.	0.
14	0.	0.	0.	0.
15	0.	0.	0.	0.
16	0.	0.	0.	0.
17	0.	0.	0.	0.
18	0.	0.	0.	0.
19	0.	0.	0.	0.
20	0.	0.	0.	0.
21	0.	0.	0.	0.
22	0.	0.	0.	0.
23	0.	0.	0.	0.
24	0.	0.	0.	0.
25	0.	0.	0.	0.
26	0.	0.	0.	0.
27	0.	0.	0.	0.
28	0.	0.	0.	0.
29	0.	0.	0.	0.
30	0.	0.	0.	0.
TOTAL	0.	48.	1125.	
AV. D.	0.0	0.7	2.9	
VOL.	0.	0.	0.	
VALUE	0.	0.	0.	
PNV=	0.0			
MARGINAL REVENUE FROM THINNING	0.0			

Fig. 24c. Periodic stand description with no thinnings displayed by the subroutine OUTPUT.

PER ACRE DATA		,SITE 110.	,IN.TREES1210.	,MANAGEMENT	1	,INT.RATE 0.03
D.B.H. CLASS		THINNING AT AGE 50.		MORTALITY FROM AGE 50. TO 55.		STAND AT AGE 55.
1		0.		0.		0.
2		0.		0.		0.
3		0.		0.		0.
4		0.		0.		0.
5		0.		0.		0.
6		0.		0.		0.
7		0.		0.		0.
8		12.		0.		0.
9		12.		0.		0.
10		12.		0.		36.
11		12.		0.		60.
12		0.		0.		24.
13		0.		0.		60.
14		0.		0.		12.
15		0.		0.		0.
16		0.		0.		0.
17		0.		0.		0.
18		0.		0.		0.
19		0.		0.		0.
20		0.		0.		0.
21		0.		0.		0.
22		0.		0.		0.
23		0.		0.		0.
24		0.		0.		0.
25		0.		0.		0.
26		0.		0.		0.
27		0.		0.		0.
28		0.		0.		0.
29		0.		0.		0.
30		0.		0.		0.
TOTAL		48.		0.		194.
AV.D.		9.1		0.0		11.2
VOL		741.		0.		4871.
VALUE		108.		0.		978.
PNV=	489.3					
MARGINAL REVENUE FROM THINNING		0.032				

Fig. 24d. Periodic stand representation under thinning displayed by the subroutine OUTPUT.

PER ACRE DATA	, SITE 110.	, IN. TREES 680.	, MANAGEMENT	1	INT. RATE 0.03
ESTABLISHMENT		OPTIMUM	PRESENT		
COST		ROTATION	VALUE		
0.		75.	713.31		
10.		75.	702.09		
20.		75.	690.86		
30.		75.	679.64		
40.		75.	668.42		
50.		75.	657.19		
60.		75.	645.97		
70.		75.	634.75		
80.		75.	623.53		
90.		75.	612.30		
100.		75.	601.08		

Fig. 24e. Present net worth and optimum rotation as a function of establishment costs displayed by the subroutine EVAL of the program FOREST.

Fig. 25. Variable-planting cost function applied to the Roseburg district in 1970.

Fig. 26. The program REGEN II flow chart.

EXPECTED PRESENT VALUE/ACRE OF A TRACT

SITE 3										
TREES/AC	COST/ACRE									
	0.	10.	20.	30.	40.	50.	60.	70.	80.	90.
XXXXXX	XXXXXX	XXXXXX	XXXXXX	XXXXXX	XXXXXX	XXXXXX	XXXXXX	XXXXXX	XXXXXX	XXXXXX
50.	494.00	483.00	472.00	461.00	450.00	440.00	429.00	418.00	407.00	396.00
170.	973.00	962.00	951.00	939.00	928.00	917.00	906.00	894.00	883.00	872.00
300.	1230.00	1219.00	1207.00	1195.00	1183.00	1172.00	1160.00	1148.00	1137.00	1125.00
680.	1408.00	1396.00	1384.00	1372.00	1361.00	1349.00	1337.00	1326.00	1314.00	1302.00
1210.	1321.00	1310.00	1298.00	1287.00	1275.00	1264.00	1253.00	1241.00	1230.00	1219.00
XXXXXX	XXXXXX	XXXXXX	XXXXXX	XXXXXX	XXXXXX	XXXXXX	XXXXXX	XXXXXX	XXXXXX	XXXXXX

Fig. 27a. Example of a production function generated by the model FOREST.

[illegible]

Fig. 27b. Treatment alternatives, densities, and cost data supplied by the user of REGEN II and displayed by the subroutine READ.

Fig. 28a. Project analysis performed by the subroutine **ALTERN**.

ROWS			COLUMNS					R (IGHT)		
ROLE	TYPE	NAME	PRIMARY ACTIVITIES				SECONDARY ACTIVITIES		H (AND)	
			Tract	Zone- elevation	Treatment	Density	Zone- elevation supplied	Zone- elevation supplier		
			a b c	I Z	I A	I D	I Z	J Z		
Objective Function.....	N	OBJ	Reduced PSV added in dollars per acre					0.		
Budget Constraint.....	≤	BDG	Reduced Cost in dollars per acre					0.		Budget
Consumption Constraints.....	=	C. IZ	Reduced Density in trees per acre					- 1.		0.
Supply Constraints.....	≤	S. JZ	0.					+ 1.		Stock in Zone Elevation JZ
Area Constraints.....	≤	A abc	+ 1.							Area of tract abc

Fig. 30. Structure of the linear programming matrix constructed by REGEN II.

Fig. 31. Subroutine ROWS flow chart.

Fig. 32. Subroutine COLUMN flow chart.

Fig. 33. Subroutine RHS flow chart.

Fig. 34. Subroutine PRELIM flow chart.

XX			
TRACT	ZONE	AREA	EFFECTIVE AREA
001	1	10.	8.0
XX			
HAND PLANTING			
		1 ACRE	TOTAL
		XXXXXX	XXXXX
COST		19.30	154.4
SEEDLINGS			
/SEEDS		300.00	2400.0
			FROM (ZONE, ELEV.)
			QUANTITY
			1 11
			1000.0 1400.0
NET VALUE			
ADDED		1051.4	8411.3

Fig. 35. Partial output of the subroutine PRELIM.

TRACT	ZONE-ELEV.	TREATMENT	DENSITY	AREA
XXXXX	XXXXXXXXXX	XXXXXXXXXX	XXXXXXXXX	XXXX
001	1	1	3	8.03
001	1	1	4	1.97
002	1	3	1	20.00
SEEDLINGS FROM ZONE-ELEV.		1	QUANTITY =	1000.00
SEEDLINGS FROM ZONE-ELEV.		11	QUANTITY =	2000.00
SEEDS FROM ZONE-ELEV.		11	QUANTITY =	15.00
003	2	3	1	40.00
003	2	3	2	10.00
SEEDS FROM ZONE-ELEV.		2	QUANTITY =	30.00
SEEDS FROM ZONE-ELEV.		11	QUANTITY =	15.00
TOTAL AREA =	80.0			

Fig. 36. The reforestation plan displayed by the subroutine FINAL.

		ZONES SUPPLIED					
		072	252	270	481	491	492
ZONE SUPPLIERS	072	X		X			
	252		X	X	X	X	
	270	X	X	X			
	481			X	X	X	
	491				X	X	X
	492						X

Fig. 37. Allowed interzone transfers of seedlings in the Roseburg district in 1970.
Crossed boxes indicate an allowed transfer.

Fig. 38. Estimated long-term real interest rates. (Source: Yohe and Karnosky, 1969).

Fig. 39. Trend of deflated Douglas-fir stumpage prices.

Fig. 40a. Value production functions for site 90.

Fig. 40b. Value production functions for site 110.

Fig. 40c. Value production functions for site 140.

Fig. 40d. Value production functions for site 170.