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Tree Volume Equations and Tables from Dendrometer Measurements

I. Tree Volume Equations from Measurements Taken with a Barr and Stroud Optical Dendrometer

Lee C. Wensel

II. Young Growth Gross Volume Tables for Sierra Redwood [*Sequoia gigantea* (Lindl.) Decne]

Lee C. Wensel and Richard L. Schoenheide

UNIVERSITY OF CALIFORNIA DIVISION OF AGRICULTURAL SCIENCES

In the first paper, the procedures are developed to compute tree volume equations from field measurements taken with a Barr and Stroud optical dendrometer. The computer programs used to perform all of the calculations are briefly described. The volume equations developed for young growth Sierra redwood are also reported, together with a discussion of the validity of these equations.

In the second paper, tree volume tables are given for young growth Sierra redwood based upon measurements taken at Mountain Home State Forest. Standard and local volume tables are given for cubic feet and Scribner board feet, together with the 95 per cent confidence intervals for the volumes in these tables. The standard table for cubic feet is based upon total height while for Scribner board feet tables are given for both merchantable height (6-inch top) and total height.

THE AUTHORS:

Lee C. Wensel is Assistant Professor of Forestry, University of California, Berkeley.

Richard L. Schoenheide is Forest Manager, Mountain Home State Forest, Springville, California.

Lee C. Wensel

I. Tree Volume Equations from Measurements Taken with a Barr and Stroud Optical Dendrometer[1]

INTRODUCTION

THE VIRTUES OF USING OPTICAL DEVICES in forest inventory have been widely documented by numerous authors. In the United States, Grosenbaugh (1963) has pioneered the use of optical devices, including the use of the Barr and Stroud dendrometer which is of concern here. Grosenbaugh (1967) has developed a computer program called STX that can be used to convert the dendrometer measurements to tree diameter, height, surface area, and volume.

Bell and Groman (1971) have reported on tests of the Barr and Stroud Type FP-12 optical dendrometer, the type used in the present study, concluding that "upper-stem diameters and segment lengths determined with the Barr and Stroud optical dendrometer are highly accurate under field conditions." They found that the accuracy in measuring tree diameters varied from 2 to 3 per cent. The accuracy in measuring stem lengths varied from about 1 per cent at elevations of about 23 feet to about 4 per cent at elevations of about 63 feet.

In connection with appropriate sampling techniques and the computer program STX (Grosenbaugh 1967), the Barr and Stroud optical dendrometer can be used to obtain volume estimates in a forest inventory *without the use of volume tables*. However, relatively few foresters have access to a Barr and Stroud dendrometer and a computer. Thus volume tables are still necessary. Even those foresters who do have access to a dendrometer still require volume tables in much of their work.

The procedures and programs presented here detail the procedures that one would use to construct standard (based upon tree DBH and height) and local (based upon tree DBH only) volume tables for any species. However, the following discussion details the application of these procedures to the tables that appear in the second paper for young growth Sierra redwood, a species for which no volume tables have previously existed.

Volume Computations

Sample trees are randomly located within the range of the tree diameters to be considered. The measurements to be collected for each tree consist of DBH measured with a diameter tape, double bark thickness at breast height, measured at two locations at right angles to each other, and dendrometer readings taken at various points on each tree. All of the data is recorded on field forms prepared by Grosenbaugh (1967) and the dendrometer readings are con-

[1] Submitted for publication January 8, 1971.

verted to tree diameters and heights by program STX. Diameter inside bark measurements were obtained using the ratio of the inside to outside bark diameters at breast height. The cubic-foot tree volumes are obtained from the "tree detail" cards punched by program STX. The necessary control cards for STX, and a sample set of data for three trees, appears in Appendix A.

The Scribner board-foot tree volumes are computed by program BFVOL (Appendix B). This program uses the "log detail" cards produced by STX[2] as its basic data and computes the Scribner board foot volume by reducing the tree to individual logs.

The merchantable height to a six-inch top, H_6, was obtained by the interpolation equation

$$H_6 = H_{d_1} + (H_{d_2} - H_{d_1})\frac{6^2 - d_2^2}{d_2^2 - d_1^2} \quad (1)$$

where d is the diameter inside bark,

$d_1 \geq 6$ and $d_2 < 6$, and H_{d_1} and H_{d_2}

are the corresponding heights.

Using the subroutine LOGS, the merchantable tree length is divided up into logs following the scaling rules used by the California Division of Forestry. Within each tree the log lengths varied from 10 to 20 feet, and were as alike as possible (see sample output from BFVOL, Appendix table B-2). Appropriate changes can be made in subroutine LOGS to use any scaling rule desired.

The scaling diameter d for each log is given by

$$d = \frac{h_1 d_2^2 + h_2 d_1^2}{h_1 + h_2} \quad (2)$$

where d_1, d_2, h_1 and h_2 are the tree diameter (inside bark) and heights, respectively, as shown in figure 1. The symbol h stands for the actual length of the log (nominal length plus trim allowance).

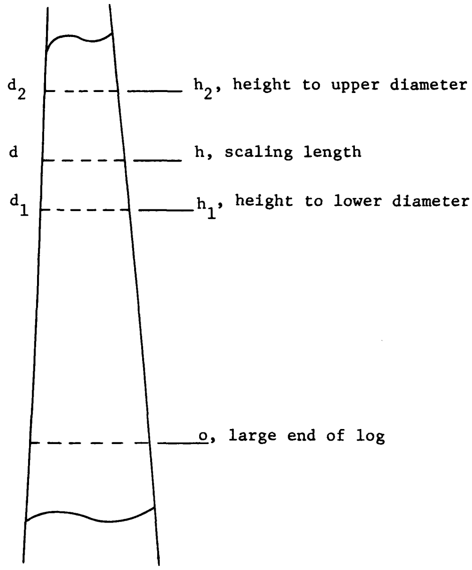

Fig. 1. Scaling diameter and length of log.

The bark thickness, b_i, needed to convert the diameter outside bark at point i, D_i, to diameter inside bark, d_i, is obtained from the relationship

$$b_i = b_{DBH}\left(\frac{D_i}{DBH}\right) \quad (3)$$

Where b_{DBH} is the bark thickness measured at breast height with a bark guage. Program STX allows for bark ratios other than (D_i/DBH) if this ratio seems inappropriate in a given application.

The Scribner board-foot volume V is calculated for each log, altering the equation for 16 foot logs (Bruce and Schumacher, 1950) by the ratio of the actual length divided by 16 feet, as follows

$$V = \frac{\text{log length}}{16}(0.79\,d^2 - 2d - 4) \quad (4)$$

Volume regressions

Once all of the volumes have been computed, the cubic-foot volumes by program STX and the Scribner board-foot volumes by program BFVOL, a

[2] STX was modified to punch an extra data card for each tree giving the diameter at the large end of the first log.

series of regression equations must be computed. The models that were chosen here have been found to give a good fit (high R^2 and well behaved at the end points) and satisfy the usual assumptions of equal variances and normal errors.

For the Sierra redwood data, program DANIEL[3] was used to fit the logarithmic form of the form factor volume equation:

$$\log V = b_0 + b_1 \log d + b_2 \log h \qquad (5)$$

to both the board-foot and cubic-foot data, where d is the tree *DBH* and h is the tree height and b_0, b_1, b_2 are the constants to be fitted. Then, recognizing the exponential relationship that exists between tree diameter and tree height (Meyer, 1940), the term *log h* was replaced by d in equation (5) to yield the local volume equation (6)

$$\log V = b_0 + b_1 \log d + b_2 d \qquad (6)$$

All logarithms are natural logarithms. i.e., to the base e.

Below are listed the coefficients for equations (5) and (6). For equation (5), which is termed a "standard" volume function because it includes terms for both tree DBH and tree height, coefficients are listed for cubic feet (total height) and Scribner board feet (merchantable height and total height). As measured by R^2, the coefficient of multiple determination, the equations all fit well, accounting for a high percentage of the total variation in each case.

Equation (5) coefficients	Cubic feet (total height)	Board feet (merch. height)	Board feet (total height)
b_0	–6.66790	–5.20645	–5.66831
b_1	1.54423	1.22817	2.21035
b_2	1.29808	1.62977	0.91921
R^2	.958	.973	.908

Equation (6) coefficients	Cubic feet	Board feet
b_0	–4.43275	–6.54770
b_1	2.85186	4.21450
b_2	–0.01911	–0.05073
R^2	.921	.888

To examine the reasonableness of the regression assumptions, the residuals are plotted by program DANIEL. For the Sierra redwood data, the results of these plots were quite similar and the plots for the cubic-foot equation (5) are given in figures 2 and 3.

Figure 2 gives the cumulative plot of the residuals on the cumulative frequency, with the ordinate axis on a logarithmic scale. To meet the necessary assumption of normality of the residuals, these data should plot approximately as a straight line through the point (0.5, 0). As evidenced by figure 2, the normality assumption appears to hold reasonably well.

Figure 3 gives the plot of the residuals on the fitted volumes for the same equation. A pattern that shows a relationship between the residuals and the fitted volumes indicates a violation of the assumptions of independence and homogeneity of variance. Since the points in figure 3 show no clear pattern, this plot supports the assumptions made.

Volume tables can then be listed using the equations developed here. The volume tables for young growth Sierra

[3] Program and instructions are available from the University of California Computer Center Library, Berkeley.

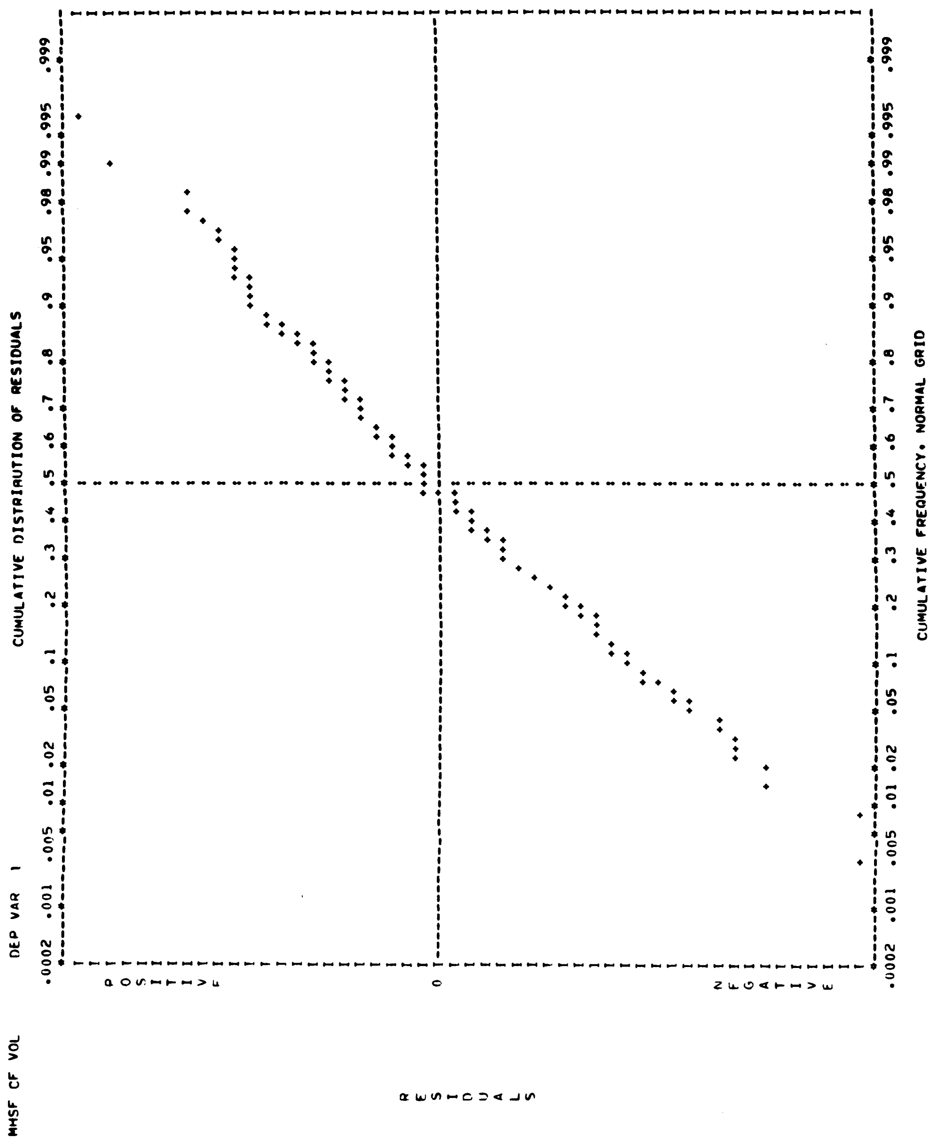

Fig. 2. Cumulative distribution for residuals for cubic foot equation (1).

redwood appear in the second paper. The computer programs used to list these tables are available from the author.

The 95 per cent confidence limits for any volume in the table can be obtained from the relationship

$$E_i = \frac{e^{\log V + 1.96\hat{\sigma}} - e^{\log V - 1.96\hat{\sigma}}}{e^{\log V}} \qquad (7)$$

which reduces to

$$E_i = e^{1.96\hat{\sigma}} - e^{-1.96\hat{\sigma}} \qquad (8)$$

where E_i is the confidence interval (in per cent) corresponding to a tree of diameter D_i at breast height and a height of H_i feet, 1.96 is the .95 probability point from the normal distribution,[4]

[4] Draper and Smith (1966), p. 121.

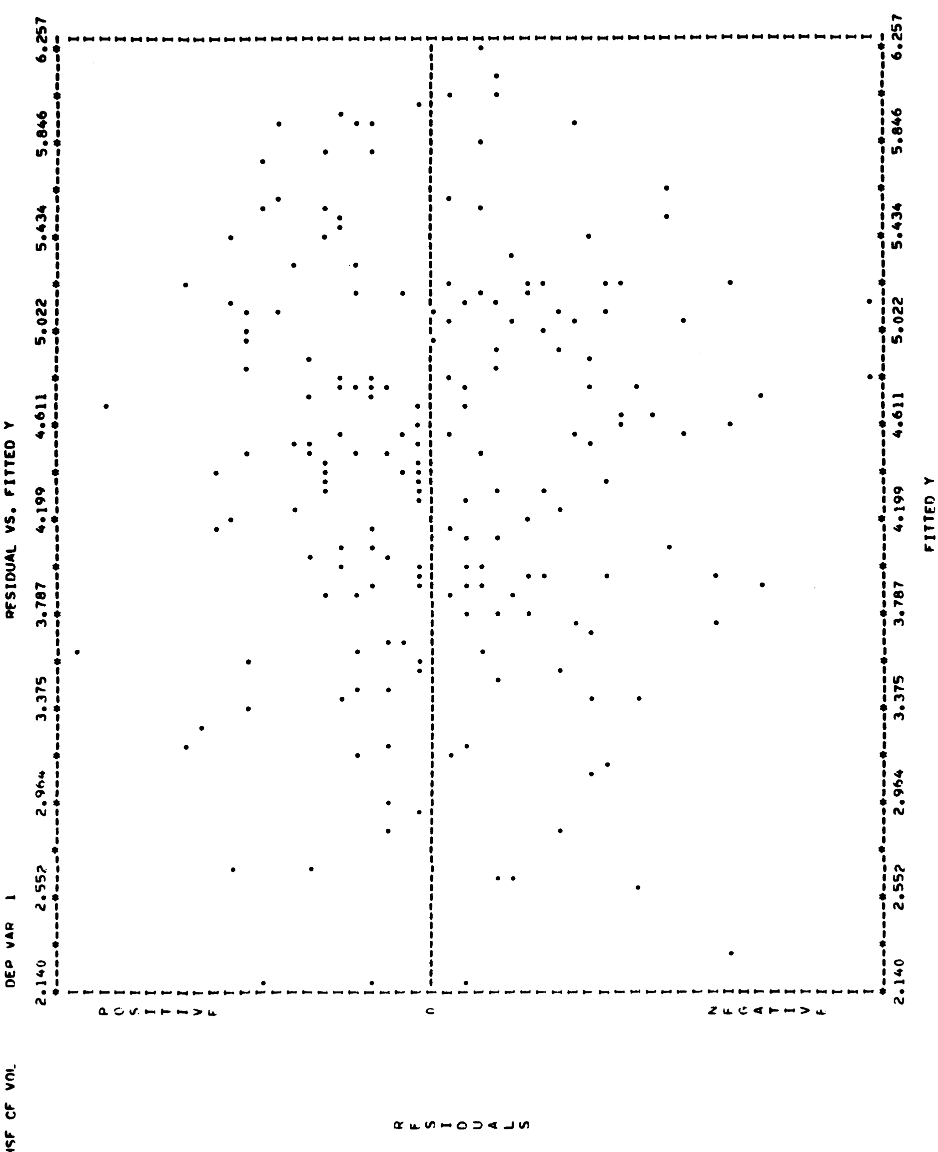

Fig. 3. Residuals on fitted volumes for cubic foot equation (1).

$$\hat{\sigma} = \text{mean square error} \sqrt{Y_i\,(X'X)^{-1}\,Y_i'} \tag{9}$$

$$X = \begin{bmatrix} 1 & \log D_1 & \log H_1 \\ 1 & \log D_2 & \log H_2 \\ \vdots & \vdots & \vdots \\ 1 & \log D_{195} & \log H_{195} \end{bmatrix}, \tag{10}$$

$$\text{and } Y_i' = \begin{bmatrix} 1 \\ \log D_i \\ \log H_i \end{bmatrix}. \tag{11}$$

The confidence interval for *log V* is symmetric ($\pm 1.96\ \hat{\sigma}$) about the volume estimate. Also, because of their independence, the variances ($\hat{\sigma}^2$) are additive, i.e., the confidence interval for $\log V_1 + \log V_2$ is $\pm\ 1.96\ \hat{\sigma}_1^2 \cdot \hat{\sigma}_2^2$.

Because of the nature of the logarithmic transformation, it is not possible to do the same thing with the untransformed values of V and $\hat{\sigma}$ and still have unbiased estimates of the confidence intervals. However, in dealing with a group of trees all about the same size, the arithmetic mean and the geometric mean (mean of logarithms) will be about the same. Under this situation, the bias would probably be small and the width of the confidence intervals for $V_1 + V_2 + \cdots + V_n$ can be estimated by

$$\pm 1.96\ n\sqrt[n]{E_1 E_2 \cdots E_n}.$$

The *exact* confidence interval is extremely complex and has never been worked out, although Naus (1969) has worked out the distribution for the sum of two log-normal variables.

APPENDIX A: Data Card Preparation for Program STX.

APPENDIX A

TABLE A-1

SAMPLE DATA CARDS FOR STX PROGRAM

```
 MOUNTAIN HOME STATE FOREST -- GIANT SEQUOIA VOLUME STUDY               MHSF   1
LCW SUMMER69 08000 01964673-11905 15658 1              77777777 123320 MHSF   2
                                                                       MHSF   3
                                                                       MHSF   4
                                                                       MHSF   5
10      =        38.6 1    2.3 2.0            60100124                 MHSF   6
11        4951446 .802    4891063 .890   489 968 .989   489 9191.072   MHSF   7
12        494 8931.175    509 8461.363   512 7881.440   517 7631.522   MHSF   8
13        521 7021.596    543 6371.676  -999-9991.764                 *MHSF   9
20      =        16.0 1    1.0 0.9            15 24 50                 MHSF  10
21        21810030.781    211 8120.974   222 7131.174   256 6461.350   MHSF  11
22       -999-9991.714                                                *MHSF  12
30      =        14.8 1     .8  .9            15 25 54                 MHSF  13
31        247 9300.811    258 8190.993   269 7381.189   304 6661.356   MHSF  14
32       -999-9991.730         M                                      *MHSF  15
```

TABLE A-2

SAMPLE OUTPUT FROM STX, TREE DETAIL CARDS USED

MOUNTAIN HOME STATE FOREST -- GIANT SEQUOIA VOLUME STUDY

DETAILED LOG AND/OR TREE REPORT

TREE/ NO./	VOLUME / CU.FT. /	SURFACE / SQ.FT. /	LENGTH / FEET /	D.I.B. / INCHES /	LOG/ CODE/	RANGE/ FEET/	TGRADS	FGRADS	SINELV
	2.0	22.4	25.9	.1		.0	-99.9	-99.9	.7640
	9.7	53.2	23.6	6.5		131.9	54.3	63.7	.6760
	8.5	32.3	9.8	10.7		110.0	52.1	70.2	.5960
	13.0	41.2	10.4	14.5		106.8	51.7	76.3	.5220
	14.7	40.1	8.7	15.8		103.1	51.2	78.8	.4400
	45.8	108.9	20.6	19.4		101.0	50.9	84.6	.3630
	24.5	54.6	9.7	21.0		91.9	49.4	89.3	.1750
	22.5	45.7	7.4	22.1		89.2	48.9	91.9	.0720
	37.3	64.3	8.8	25.0		89.2	48.9	96.8	-.0110
	81.8	92.4	8.5	30.6		89.2	48.9	106.3	-.1100
	0.	0.	0.	52.6		92.4	49.5	144.6	-.1980
1	260.2	555.1	133.4	38.6					
	2.4	24.6	26.2	.1		.0	-99.9	-99.9	.7140
	2.7	16.4	8.0	7.1		43.4	25.6	64.6	.3500
	4.4	21.3	8.2	8.7		41.3	22.2	71.3	.1740
	8.1	28.3	7.9	11.1		40.7	21.1	81.2	-.0260
	0.	0.	0. 1	6					
	0.	0.	0.	16.2		41.1	21.8	100.3	-.2190
2	17.6	90.6	50.4	16.0					
	3.3	30.7	30.3	.1		.0	-99.9	-99.9	.7300
	3.4	18.9	8.4	7.6		47.2	30.4	66.6	.3560
	5.3	23.9	8.7	9.5		44.3	26.9	73.8	.1890
	7.3	26.6	7.8	11.6		43.5	25.8	81.9	-.0070
	0.	0.	0.	14.6		42.8	24.7	93.0	-.1890
3	19.2	100.1	55.2	14.8					

TABLE A-3

SAMPLE PUNCHED CARD OUTPUT FROM STX, TREE DETAIL CARDS USED AS INPUT TO BFVOL

(1) (2)	(3) (4)	(5) (6)	(7)	(8) (9)			
11111	38.6133.4	.1133.4	2.0	22.4 25.9	1.000	3.7MHSF	1
11011	38.6133.4	6.5107.5	9.7	53.2 23.6	1.000	51.2MHSF	2
1 911	38.6133.4	10.7 83.8	8.5	32.3 9.8	1.000	54.5MHSF	3
1 811	38.6133.4	14.5 74.1	13.0	41.2 10.4	1.000	88.6MHSF	4
1 711	38.6133.4	15.8 63.7	14.7	40.1 8.7	1.000	104.5MHSF	5
1 611	38.6133.4	19.4 55.0	45.8	108.9 20.6	1.000	335.8MHSF	6
1 511	38.6133.4	21.0 34.4	24.5	54.6 9.7	1.000	182.3MHSF	7
1 411	38.6133.4	22.1 24.7	22.5	45.7 7.4	1.000	169.8MHSF	8
1 311	38.6133.4	25.0 17.3	37.3	64.3 8.8	1.000	290.0MHSF	9
1 211	38.6133.4	30.6 8.5	81.8	92.4 8.5	1.000	669.1MHSF	10
1 111	38.6133.4	52.6 0.	0.	0. 0.	1.000	0. MHSF	11
2 511	16.0 50.4	.1 50.4	2.4	24.6 26.2	1.000	5.6MHSF	12
2 411	16.0 50.4	7.1 24.2	2.7	16.4 8.0	1.000	13.1MHSF	13
2 311	16.0 50.4	8.7 16.2	4.4	21.3 8.2	1.000	24.9MHSF	14
2 211	16.0 50.4	11.1 7.9	8.1	28.3 7.9	1.000	53.4MHSF	15
2 111	16.0 50.4	16.2 0.	0.	0. 0.	1.000	0. MHSF	16
3 511	14.8 55.2	.1 55.2	3.3	30.7 30.3	1.000	9.1MHSF	17
3 411	14.8 55.2	7.6 24.9	3.4	18.9 8.4	1.000	17.5MHSF	18
3 311	14.8 55.2	9.5 16.5	5.3	23.9 8.7	1.000	30.9MHSF	19
3 211	14.8 55.2	11.6 7.8	7.3	26.6 7.8	1.000	47.0MHSF	20
3 111	14.8 55.2	14.6 0.	0.	0. 0.	1.000	0. MHSF	21

(1) Tree number
(2) Log number
(3) DBH
(4) Total height of tree in feet
(5) Diameter inside bark in inches
(6) Height above base of tree in feet
(7) Cubic foot volume of log
(8) Surface area in square feet
(9) Log length in feet

TABLE A-4

SAMPLE OUTPUT FROM PROGRAM STX, TREE DETAIL CARDS USED FOR REGRESSION

(1)		(2)	(3)		(4)	(5) (6)		(7)	(8)	
111	111	38.6	4.3	-0	260.2	555.1133.4	1.000	8.126	1949MHSF	1
211	111	16.0	1.9	-0	17.6	90.6 50.4	1.000	1.396	97MHSF	2
311	111	14.8	1.7	-0	19.2	100.1 55.2	1.000	1.195	104MHSF	3

(1) Tree number
(2) DBH*
(3) Double bark
(4) Vol cu ft*
(5) Surface area
(6) Height*
(7) BA
(8) Vol bd ft

* Used in regression by program DANIEL.

APPENDIX B: Program BFVOL

APPENDIX B

TABLE B-1

PROGRAM BFVOL LISTING. PROGRAM COMPUTES THE SCRIBNER BOARD FOOT VOLUMES FROM TREE DETAIL INFORMATION OBTAINED FROM PROGRAM STX

```
      PROGRAM BFVOL(INPUT,OUTPUT,PUNCH)
      DIMENSION DIB(20), TL(20)
      DIMENSION D(20), DBH(196), HMER(196), IBV(196), N(196)
      COMMON /LOGS/ L(10),NLOGS,TREEL,LOGL
      PRINT 190
      K=1
      I=0
   10 I=I+1
      READ 140, N(K),NB,DBH(K),DIB(I),TL(I)
      IF (N(K).EQ.1000) GO TO 120
      IF (NB.NE.1) GO TO 10
C
C     TO FIND MERCH. HEIGHT WITH THE TOP OF 6-INCH DIB.
      J=1
   20 IF (DIB(J).LT.6..AND.DIB(J+1).GE.6.) GO TO 30
      J=J+1
      GO TO 20
   30 IF (DIB(J+1).EQ.6.) GO TO 40
      TREEL=TL(J)-(TL(J)-TL(J+1))*((36.-DIB(J)**2)/(DIB(J+1)**2-DIB(J)**
     12))
      GO TO 50
   40 TREEL=TL(J+1)
   50 CALL LOGS
C
C     TO FIND SCALING DIAMETERS.
      NL=NLOGS
      M=1
      SUM=0.
   60 SUM=SUM+L(NL)+0.5
   70 IF (SUM.GE.TL(I).AND.SUM.LT.TL(I-1)) GO TO 80
      I=I-1
      GO TO 70
   80 IF (SUM.EQ.TL(I)) GO TO 90
      D(M)=SQRT(((TL(I-1)-SUM)*DIB(I)**2+(SUM-TL(I))*DIB(I-1)**2)/(TL(I-
     11)-TL(I)))
      GO TO 100
   90 D(M)=DIB(I)
  100 NL=NL-1
      M=M+1
      IF (NL.NE.0) GO TO 60
C
C     TO FIND BOARD FOOT VOLUME
C     V = (.79DSQ - 2D - 4) * L/16.
      V=0.
      JJ=NLOGS
      DO 110 II=1,NLOGS
      V=V+(.79*D(II)**2-2.*D(II)-4.)*L(JJ)/16.
      JJ=JJ-1
  110 CONTINUE
      IBV(K)=V
      HMER(K)=TREEL
      IF (K.EQ.51.OR.K.EQ.101.OR.K.EQ.151) PRINT 190
      PRINT 180, N(K),TREEL,LOGL,NLOGS,(L(J),J=1,NLOGS)
      I=0
      K=K+1
      GO TO 10
  120 K=K-1
      PRINT 150
      DO 130 I=1,K
      IF (K.EQ.51.OR.K.EQ.101.OR.K.EQ.151) PRINT 150
      PRINT 160, DBH(I),HMER(I),IBV(I),N(I)
      PUNCH 170, DBH(I),HMER(I),IBV(I),N(I)
  130 CONTINUE
      STOP
```

APPENDIX B—TABLE B-1 *Continued*

```
C
C
  140 FORMAT (I4,I2,10X,F4.1,5X,2F5.1)
  150 FORMAT (1H1, 14X, 6HMERCH., 4X, 6HBD9FT9, 5X, 4HTREE/ 7X, 3HDBH,
     1  5X, 6HHEIGHT, 4X, 6HVOLUME, 6X, 3HNO./)
  160 FORMAT (2F10.1, 2I10)
  170 FORMAT (10X,2F10.1,20X,I10,10X,I10)
  180 FORMAT (5X,I3,3X,F8.1,I8,1X,I6,3X,10I4)
  190 FORMAT (1H1,/5X,4HTREE,5X,21HMERCH.  SCALE     NO./6X,3HNO., 5X,38
     1HHEIGHT   LENGTH    LOGS    LOG LENGTHS,/)
      END

      SUBROUTINE LOGS
      COMMON /LOGS/ L(10),NLOGS,TREEL,LOGL
C
C     ---- LOG BREAK DOWN FOR SCALING, GIVEN MERCH.LENGTH (TREEL)
C
      DO 10 I=1,10
   10 L(I)=0.
C
C     LOGS TRUNCATED TO EVEN FEET (CHANGE 2.5 TO 0.5 TO ROUND)
      NLOGS=INT((TREEL-2.5)/20.5)+1
      IF (NLOGS.GT.10) PRINT 130, NLOGS
      LOGL=INT((TREEL-NLOGS*0.5)/2.)*2
      IF (NLOGS.NE.1) GO TO 20
      L(1)=LOGL
      GO TO 110
   20 L(1)=(LOGL/NLOGS/2)*2
      IF (L(1).EQ.20) GO TO 30
      IF (NLOGS*L(1).NE.LOGL) GO TO 50
C
C     ALL LOGS ARE THE SAME LENGTH.
   30 DO 40 I=2,NLOGS
   40 L(I)=L(1)
      GO TO 110
   50 N=NLOGS-1
C
C     UPON EXIT FROM THIS LOOP (BRANCH TO 60) (I) WILL
C     EQUAL THE NUMBER OF SHORTER LOGS.
      DO 60 I=1,N
      IF (NLOGS*L(1).EQ.(LOGL-2*I)) GO TO 70
   60 CONTINUE
C
C     ERROR IN COMPUTATIONS, LENGTHS NOT RESOLVED.
      PRINT 120, NLOGS,LOGL,L(1)
      GO TO 110
   70 IF (I.EQ.1) GO TO 90
C
C     SET ALL SHORTER LOGS EQUAL TO L(1).
      DO 80 J=2,I
   80 L(J)=L(1)
   90 I=I+1
C
C     SET ALL LONGER LOGS EQUAL TO L(1)+2.
      DO 100 J=I,NLOGS
  100 L(J)=L(1)+2
  110 RETURN
C
  120 FORMAT (32HERROR IN CALCULATING LOG LENGTHS,/5X,7HNLOGS =,I3,5X,/6
     1HLOGL =,I3,5X,6HL(1) =,I3)
  130 FORMAT (23H TOO MANY LOGS, NLOGS =,I4)
      END
```

TABLE B-2

BFVOL OUTPUT. LISTING OF TREE VOLUME AND HEIGHT INFORMATION

Tree number	DBH	Merch. height*	Scale length	Logs	Log length						Merch. volume
	inches	*feet*	*feet*	*number*	*feet*						*bd. ft.*
1	38.6	111.3	108	6	18	18	18	18	18	18	1,306
2	16.0	31.7	30	2	14	16					49
3	14.8	36.3	34	2	16	18					62

* Used in regression models along with DBH.

Acknowledgments

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LITERATURE CITED

BELL, J. F. and W. A. GROMAN

1971. A field test of the accuracy of the Barr and Stroud Type FP-12 optical dendrometer. Forestry Chronicle **47**(2):69–74. Canadian Institute of Forestry.

BRUCE, D. and F. X. SCHUMACHER

1950. Forest Mensuration. New York: McGraw-Hill Book Company.

DRAPER, N. R. and H. SMITH

1966. Applied regression analysis. New York: John Wiley and Sons, Inc.

GROSENBAUGH, L. R.

1963. Optical dendrometers for out of reach diameters: a conspectus and some new theory. Forest Science Mono. **4**. Society of American Foresters, Wash., D.C.

1967. STX—Fortran 4 program for estimates of tree populations from 3P sample-tree-measurements. U. S. Forest Service, Berkeley: Research Paper PSW-**13**, revised.

MEYER, H. A.

1940. A mathematical expression for height curves, Jour. of Forestry **38**(5):415-20. Society of American Foresters, Wash., D.C.

NAUS, J. I.

1969. The distribution of the logarithm of the sum of two log-normal variates. Jour. of the American Statistical Assn. **64**(326):655–59.

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