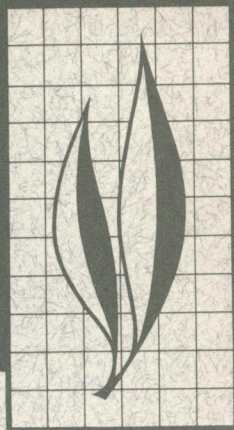


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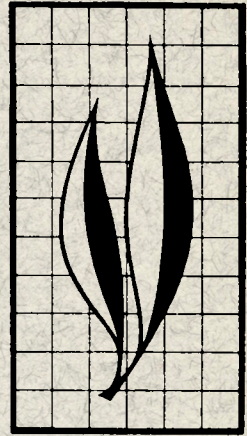
## Influence of Particle Size Distribution in Soil Compaction

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The theory and experimentation reported are for the purpose of determining the effect of particle size proportions in soils on their maximum bulk densities and other properties affected by compaction. If the different size classes in a bidisperse or tridisperse mixture have very large effective diameter ratios and if certain other simplifying requirements are followed, it is shown theoretically that the minimum bulk volume of the mixtures can be predicted from the volume proportions of the different-sized particles present. The minimum bulk volumes and hence maximum bulk densities of two prepared textural sequences, Stockton silty clay and Monterey sand mixtures, were obtained after kneading compaction at different water contents and were examined with respect to the above theory. The relationships between water content and compacted bulk density, between water content and suction of compacted soils, and the formally calculated mean effective pore diameter distribution at minimum and maximum bulk density are presented and discussed.

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## **Influence of Particle Size Distribution in Soil Compaction<sup>1</sup>**

### **INTRODUCTION**

SOIL COMPACTION is the process of bringing solid soil particles closer together. When such rearrangement occurs, the bulk volume of the soil is said to diminish and the bulk density to increase. Soil bulk density is commonly used as a measure of soil compaction.

Compaction reduces the gross pore space and may be expected to cause a new frequency distribution of effective pore sizes. Conceivably, a quantity of solid particles with a suitable geometric shape might be rearranged by compaction so that all voids are destroyed, but it is highly improbable that such a condition would ever occur in nature. The specific bulk volume of soil, however, may conveniently be thought of as lying somewhere between a lower limit equal to the specific particle volume (void ratio = zero) and an upper limit that is rarely more than about two and one-half times as great (void ratio = 1.5).<sup>2</sup> Between these limits the exact volume

that a given mass of soil will occupy is greatly influenced by its recent history. Evidently the amount of further possible change in specific bulk volume of a given soil, and the work required to produce additional change, depend on the magnitude of this volume with respect to the stated limits; in general, the smaller the void ratio, the more difficult is yet further reduction (Bodman and Rubin, 1948).

It is reasonable to suppose that, among other soil properties, particle size distribution will influence the minimum bulk volume obtainable under given conditions of compaction. The purpose of this paper is to present evidence concerning the influence of particle size distribution on soil compaction and on some of the properties of compacted soil mixtures. It includes no analysis of the forces causing compaction.

### **BULK VOLUMES OF SIMPLIFIED PARTICULATE SYSTEMS**

When two lots of water-free, monodisperse, noninteracting solid particles are mixed, each lot being of any uniform

particle shape and possessing its own characteristic packing pattern and effective particle size, the newly created

<sup>1</sup> Submitted for publication February 2, 1965.

<sup>2</sup> When considering a compressible particulate body such as soil the void ratio has a distinct advantage over the porosity as a quantitative expression of pore space. In a given body of soil material the void ratio refers the volume of pores (non-solids) to the constant, solid-particle volume, whereas the porosity refers the pore volume to the variable bulk volume. If  $e$  = void ratio and  $E$  = porosity,  $E = e/(1 + e)$ . The desirability of using the void ratio rather than the porosity is analogous to the desirability, in most soils studies, of expressing soil water content on the dry rather than on the wet basis.



bidisperse (binary) mixture will possess a bulk volume that generally is less than the sum of the original bulk volumes of the two monodisperse systems. The greater the difference in effective particle size of the two components, the greater will be the reduction in bulk volume brought about by mixing. The reduction also will depend upon the proportions of the different size classes present and their respective packing peculiarities. The limiting difference for a binary mixture in any given proportions will be obtained when the effective diameter ratio between the two size classes is sufficiently great that boundary effects may be neglected. This reasoning may be extended to tridisperse (ternary) mixtures and polydisperse mixtures of a higher order. If the packing pattern and total volume of solid particles of each size class present are clearly defined, and certain simplifying assumptions are made, it is possible to introduce an orderly understanding of the volume changes that occur upon mixing. Early experimental and explanatory studies of such changes were made by Westman and Hugill (1930), and Westman (1936) with binary and ternary mixtures of small spheres. Furnas and others have done similar work on the packing of large, irregularly shaped particles (reviewed by DallaValle, 1943).

In order to simplify consideration of the relation between particle size distribution and bulk volume of polydisperse mixtures created from different-sized monodisperse particles, certain assumptions will be made: (1) the particles do not interact; (2) the particles of a given size class may be considered homogeneous with respect to effective shape, whatever that shape may be; (3)

effective size, shape, and abundance are unaltered by mixing; (4) when mixed, the geometrical boundary effects at particle surfaces, both within the mixtures and at the outer borders, are negligible;<sup>3</sup> (5) the packing pattern of a single size class is definable by the void ratio characteristic of the class concerned at its minimum bulk volume and persists within the matrix (see p. 569) or any continuous assemblage of that size class, after mixing; (6) any mixing process may be used provided it does not affect the other requirements. Only minimum bulk volumes will be considered in the resultant mixtures. This requirement is clearly necessary since, for a given mixture, none but the minimum obtainable bulk volume and void ratio is unique. Bulk volumes other than the minima could be made to diminish upon further mixing or upon more careful placement of individual particles.

The notched, triangular prism represented isometrically in figure 1A indicates minimum bulk volumes and void ratios of all tridisperse systems that may be produced by thorough mixing, in any proportions, of three monodisperse components which possess certain definite packing characteristics and fulfill the six specifications stated in the preceding paragraph. The apices of the equilateral base represent the monodisperse systems themselves and are labeled *c* (coarse), *m* (medium), and *f* (fine). These letters are also used as identifying subscripts in the discussion. Triangular coordinates are used in the conventional way on the base for defining the particle volume composition of a mixture, so that the sum of solids volumes of all different particle size classes present equals unity for any single mixture, that is,

$$V_{sc} + V_{sm} + V_{sf} = 1 \quad [1]$$

<sup>3</sup> This is equivalent to requiring that internal boundary effects be negligible in a system of infinite extent.



The minimum bulk volume for any one mixture is shown by the vertical distance from its volume composition point to one of the oblique plane surfaces above. Thus the solids volumes ( $V_{sc}$ ,  $V_{sm}$ ,  $V_{sf}$ ), the void ratios ( $e_c$ ,  $e_m$ ,  $e_f$ ), and minimum bulk volumes ( $1 + e_c$ ,  $1 + e_m$ ,  $1 + e_f$ ) of the three different monodisperse systems are shown along the three vertical edges of the prism, at  $c$ ,  $m$  and  $f$ . Except at the apices and sides the vertical representing the bulk volume of a ternary mixture is  $V_{bcmf} =$

$1 + e_{cmf}$  in which  $e_{cmf}$  = mean void ratio of a definite mixture of  $c$ ,  $m$  and  $f$ , and  $V_{bcmf}$  = minimum bulk volume of the same mixture. The three sides of the prism graphically portray the bulk volumes and void ratios of all members of the three kinds of binary mixtures, ( $c + m$ ), ( $m + f$ ), and ( $f + c$ ). The actual conformation of the notched top of the prism depends upon the particular void ratio and minimum bulk volume of an assemblage of each size class when packed alone (that is, each of the three possible monodisperse assemblages). Construction of a scale diagram for general discussion, therefore, requires the choice of definite, arbitrary values for void ratios. Those chosen for figure 1A were:  $e_c = 0.5$ ,  $e_m = 0.8$ ,  $e_f = 1.0$ . Other ratios of reasonable magnitude, of which any two or all three could have been equal, would have served equally well for illustration.

Construction of the prism generally must be based upon linear equations which, in turn, depend upon requirements (1) to (6) and interrelate the solid-volume proportions of the different size-class components with bulk volume and void ratio of binary and ternary systems. The most important equations are introduced in the following discussion.

The term "matrix" is useful. A matrix of any uniform size class,  $i$ , will be said to exist in a mixture of different size classes at the minimum bulk volume, if the volume contribution of that particular size class to the bulk volume of the mixture is given by the product  $V_{si}(1 + e_i)$  in which  $e_i$  and  $V_{si}$  are, respectively, the void ratio and particle volume of the size class  $i$ . For this condition the bulk volume of size class  $i$  will form a matrix of characteristically packed, tangent particles of class  $i$  in which particles of the other size classes will be embedded. The embedded par-

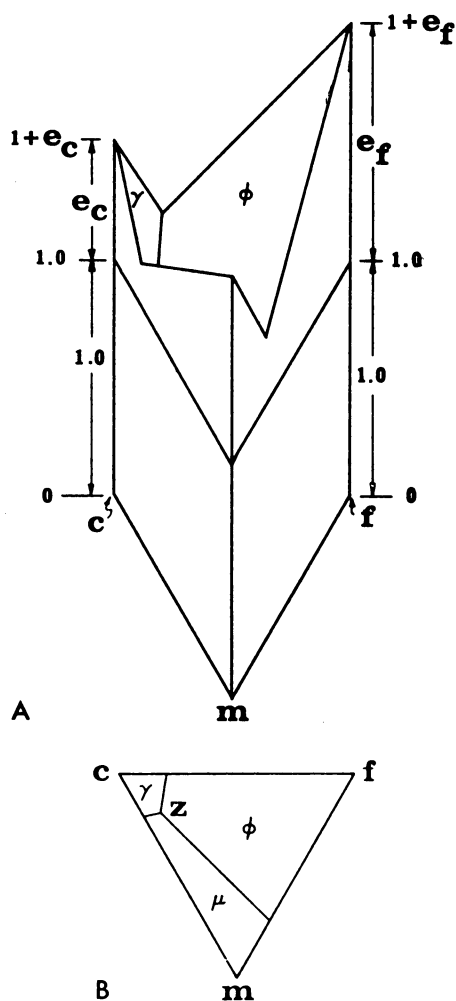


Fig. 1. A. Minimum bulk volumes and void ratios of ideal ternary mixtures. B. Matrix transition lines of ideal ternary mixtures projected on composition base.



ticles, on the other hand, do not touch so as to form a continuous matrix of characteristic packing pattern and void ratio except when the mixture contains certain critical volume proportions of solids of different size classes. Abrupt changes occur in the minimum bulk volume-composition trends as one matrix transforms to another. It is evident that either medium or fine particles or both can occupy the pores in a matrix of coarse; that neither coarse nor medium particles can occupy the pores in a matrix of fine, although they may be present among (that is, embedded by) the fine; and that fine particles are able, but coarse particles are unable, to occupy the pores in a matrix of medium.

The notches in the prism of figure 1A

are produced by three intersecting, oblique planes that indicate decreases in  $e_{cmf}$  and  $V_{bcmf}$  toward the center. All planes may be seen in the plan view but parts of only two are visible in the elevation of figure 1A:  $\gamma$  intersecting the edge above  $c$  at  $V_{bc} = (1 + e_c)$ , and  $\phi$  intersecting the edge above  $f$  at  $V_{bf} = (1 + e_f)$ . The  $\mu$  plane is hidden, but its point of intersection with the edge above  $m$  is visible at  $V_{bm} = (1 + e_m)$ . The three planes are the loci of minimum bulk volumes for which matrices of coarse ( $\gamma$  plane), medium ( $\mu$  plane), and fine ( $\phi$  plane), respectively, exist in the mixtures. The equations for the matrix planes are, within the limits indicated,

$$\gamma: V_{bcmf} = V_{sc}(1 + e_c) \quad [2]$$

$$\mu: V_{bcmf} = V_{sm}(1 + e_m) + V_{sc} \quad [3]$$

$$\phi: V_{bcmf} = V_{sf}(1 + e_f) + V_{sc} + V_{sm} \quad [4]$$

The void ratios of the mixtures ( $e_{cmf}$ ) at any point in each plane may be obtained by subtracting the sum of the volumes of solid particles ( $= 1$ ) from the bulk volume of the mixture in the appropriate equation. Intersections of the planes create inward-sloping "troughs" and the straight lines of intersection at trough bottoms mark sharp transitions from one matrix condition to another. If images of the straight lines of intersection are vertically projected on the base (figure 1B), volume composition lines are traced out that correspond to the innumerable com-

positions at which transition occurs. Two coexistent matrices are present along each of the transition lines. There is a "triple point" (labeled  $z$  in plan) at the particle volume composition corresponding to the absolute minimum bulk volume. Matrices of each of the three size classes coexist at this point and are, of course, completely interpenetrating.

The proportions of two components of those ternary mixtures that can pack to give the matrix transitions  $\gamma/\mu$  and  $\mu/\phi$  are,

$$\gamma/\mu \text{ transition: } \frac{V_{sc}}{V_{sm}} = \frac{(1 + e_m)}{e_c} \quad [5]$$

$$\mu/\phi \text{ transition: } \frac{V_{sm}}{V_{sf}} = \frac{(1 + e_f)}{e_m} \quad [6]$$

Since the  $e$  values in [5] and [6] are constants, the proportions of two components in each of these two transitions also are constant until the absolute

minimum bulk volume composition is reached (point  $z$ , fig. 1B) where the third component also forms a matrix.

For the  $\phi/\gamma$  transition:



$$V_{sf} = V_{sc} \frac{(1 + e_c)}{e_f} - \frac{1}{e_f} \quad [7]$$

But at  $V_{sm} = 0$  and nowhere else,

$$\frac{V_{sf}}{V_{sc}} = \frac{e_c}{(1 + e_f)} \quad [7A]$$

It is evident that a constant ratio  $V_{sf}/V_{sc}$  does not exist along the  $\phi/\gamma$  transition line [7] although it, too, leads to a single matrix of the third com-

ponent beyond the point of absolute minimum bulk volume. The triangular coordinates for the absolute minimum bulk volume are

$$V_{sc} = \frac{1}{a}(1 + e_m)(1 + e_f) \quad [8]$$

$$V_{sm} = \frac{1}{a}(1 + e_f)e_c \quad [9]$$

$$V_{sf} = \frac{1}{a}(e_me_c) \quad [10]$$

in which

$$a = [(1 + e_m + e_c)(1 + e_f) + e_me_c]$$

Recalling the void ratios ( $e_c = 0.5$ ,  $e_m = 0.8$ ,  $e_f = 1.0$ ) assumed for figure 1A, we obtain, for our illustration, the following values at the absolute minimum bulk volume: particle volume composition (equations [8], [9], [10])

$$V_{sc} = 0.72; V_{sm} = 0.20; V_{sf} = 0.08,$$

bulk volume (equations [2], [3], or [4])

$$V_{bcmf} = 1.08,$$

mean void ratio<sup>4</sup>

$$e_{cmf} = (V_{bcmf} - 1) = 0.08$$

Calculation of the mean bulk density for any mixture requires knowledge of the mean particle density of each size class. At the absolute minimum bulk volume the mean bulk density is given by

$$\rho_{bcmf} = \frac{V_{sc}\rho_{sc} + V_{sm}\rho_{sm} + V_{sf}\rho_{sf}}{V_{bcmf}} \quad [11]$$

in which  $\rho_s$  represents particle density, in units consistent with  $\rho_b$ . If, in the illustration, the solid particle volumes are expressed in cubic centimeters,  $V_{scmf} = 1000 \text{ cm}^3$ ,  $\rho_{sc} = \rho_{sm} = \rho_{sf} = 2.65 \text{ g/cm}^3$ , and  $V_{bcmf} = 1080 \text{ cm}^3$ , bulk density at absolute minimum bulk volume,  $\rho_{bcmf} = 2.45 \text{ g/cm}^3$ .

Intersection of the matrix surfaces of figure 1A by a horizontal plane, placed normal to the prism axis, produces contour lines giving the positions

<sup>4</sup>It is entirely by chance that void ratios were chosen so that  $V_{sf}$  and  $e_{cmf}$  turn out to be numerically equal.



of equal minimum bulk volumes. If projected on the base, the contours reveal the very great number of mixtures having different particle size distributions but the same minimum bulk volume and void ratio. This is also clearly shown by equations [2], [3], and [4]. In a given set of mixtures obeying the same packing rules, each of which conforms in behavior to an ideal ternary system, there is, therefore, no unique connection be-

tween particle size distribution and minimum bulk volume or void ratio, except for the absolute minimum bulk volume and its void ratio which can be produced only by a single mixture of definite size-class proportions (equations [8], [9], and [10]). Further examination of equations [1], [2], [3], and [4] adds to our general information about the contour lines, and it is seen that:

along the  $\gamma$  matrix contours,  $dV_{sc} = 0$ ,  $dV_{sm} = -dV_{sf}$

along the  $\phi$  matrix contours,  $dV_{sf} = 0$ ,  $dV_{sc} = -dV_{sm}$

along the  $\mu$  matrix contours,  $dV_{sm} = \frac{1}{e_m} dV_{sf}$  and

$$dV_{sc} = -\frac{(1 + e_m)}{e_m} dV_{sf}$$

The same minimum bulk volume may be produced by only two of each of the three kinds of *binary* mixtures (consider a horizontal line drawn above notch bottom on each of the prism

faces) except for the single minimum bulk volume corresponding to the matrix transition value, for which there is only one composition.

## PACKING OF REAL PARTICLES

Equations [2] to [10] and the model in figure 1 give the packing relationships for systems that rigorously fulfill the specified requirements. There are differences when the concepts involved are carried over to soils and other systems of real particles. The basic requirements that concern boundary influences and particle interaction then need special attention.

Regardless of particle shape, complete avoidance of boundary effects in collections of noncoherent, noninteracting particles is possible only if effective diameter ratios are indefinitely large—that is, if  $d_m/d_n \rightarrow \infty$ , in which  $d_m$  and  $d_n$  are the diameters of the larger and the smaller of any two size-adjacent classes. Boundary effects, both at particle surfaces and container walls, may cause noticeable departure from the ideal relations of figure 1A when experi-

ments are made with size classes having finite diameter ratios. The departures become more conspicuous as  $d_m/d_n$  diminishes. Polydisperse systems that occur naturally, such as soils and unsorted sediments, commonly have fairly continuous particle-size representation. Diameter ratios of adjacent size classes will therefore be finite and small in most soils and sediments, and deviations from idealized packing may be considerable.

Interaction between particles of certain different size classes will not necessarily prevent an approach to the idealized ternary relations, provided that diameter ratios are large and that interaction fails to interfere with the creation of a minimum bulk volume equal to that which would occur in the absence of interaction. An example would

be weak and temporary bonding that, owing to its destructibility, would not prevent thorough mixing and packing as described. Uniformly sized, strongly bonded clusters of smaller particles behaving as single larger particles might also offer little interference. Such clusters may be thought of as comprising a single size class in idealized binary and ternary systems, provided that the clus-

ters persist, preserve their characteristic void ratios during mixing, and meet all remaining specifications. Each complex component will then, of course, contribute a quota of fine, internal pore space in addition to the usual quota of coarser, inter-aggregate pore space, the latter being characteristic of the packing pattern of aggregates when in their customary tangent assembly, *en masse*.

## EXPERIMENTAL

### Binary Mixtures of Noncoherent Particles

Two series of binary mixtures, A and B, were prepared from monodisperse, noncoherent particles of sand and small, solid glass spheres. After mixing, minimum void ratios ( $e_{ob}$ ) were measured for comparison with the theoretical minimum void ratios ( $e_{th}$ ) determined from equations [7A], [1], and [2]. Results are given in table 1 and are plotted in figure 2.

The monodisperse systems were prepared and their void ratios measured before mixing, in the same way as the different binary mixtures. Mixing and compacting consisted of vigorously jar-

ring a vessel containing the coarse or fine particles, alone or as mixtures, until no further reduction in bulk volume could be seen. There was no floating of one class upon the other.

The measured void ratios of the monodisperse systems, fine and coarse in each series, respectively, were taken as providing the packing patterns and void ratios for the corresponding component matrices in the binaries. This is in accord with our required basic assumption and is implicit in calculation of all theoretical void ratios entered in table 1. It may be pointed out that when void

TABLE 1  
VOID RATIOS OF SOME BINARY MIXTURES AT THEIR  
MINIMUM BULK VOLUMES\*

A. Coarse beads and fine beads $d_c/d_f = 11$				B. Coarse sand and fine beads $d_c/d_f = 41$			
Volume ratio, $c/f$	$e_{th}$	$e_{ob}$	Relative deviation	Volume ratio, $c/f$	$e_{th}$	$e_{ob}$	Relative deviation
0/100.....		0.632		0/100.....		0.632	
37.5/62.5.....	0.395	0.404	+0.023	36.6/63.4.....	0.401	0.400	-0.002
47.4/52.6.....	0.332	0.358	+0.078	46.4/53.6.....	0.339	0.351	+0.035
57.5/42.5.....	0.269	0.305	+0.134	56.5/43.5.....	0.275	0.287	+0.044
68.0/32.0.....	0.202	0.275	+0.361	66.9/33.1.....	0.209	0.242	+0.158
71.9/28.1†.....	0.177			72.2/27.8.....	0.176	0.225	+0.278
72.0/28.0.....	0.179	0.277	+0.547	73.7/26.3†.....	0.167		
78.3/21.7.....	0.282	0.319	+0.131	77.8/22.2.....	0.232	0.275	+0.185
100/0.....		0.637		100/0.....		0.583	
Sum.....			+1.274	Sum.....			+0.700
Mean.....			+0.212	Mean.....			+0.116

\* Effective diameters and densities of particles:

	DIAMETER	PARTICLE DENSITY
Coarse sand (quartz).....	1 < 1.2 mm	2.614 gm/cm <sup>3</sup>
Coarse beads (solid glass).....	0.30	2.508
Fine beads (solid glass).....	0.027	2.261

† Critical volume ratios corresponding to absolute minimum bulk volumes.



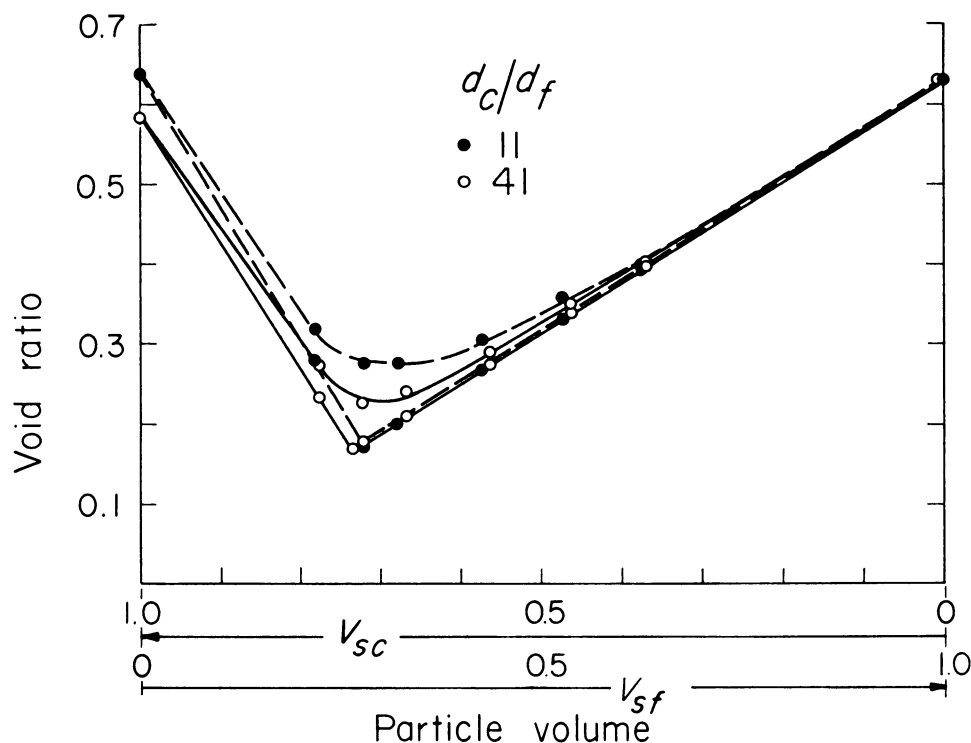


Fig. 2. Minimum void ratios of binary mixtures of sand and beads, observed and theoretical, showing effect of diameter ratio.

ratios and/or bulk volumes are plotted in relation to volume composition of the binary (figure 2), the graph corresponds to the vertical, coarse-plus-fine face of the figure 1A prism.

The two series consisted of particles having the diameter ratios,  $d_c/d_f$  (series A) = 11 and  $d_c/d_f$  (series B) = 41. It was possible, therefore, to examine the effect of the ratio differences on the relative deviation  $(e_{ob} - e_{th}) / e_{th}$ .

Table 1 indicates the marked increases obtained between observed and theoretical void ratios as the absolute minimum void ratio is approached from either side. Thus, at about 37 per cent fine particles, relative deviations are less than 0.025 (2.5 per cent) in both series. At a fine-particle content of about 28 per cent, a composition corresponding closely to the absolute minimum for the mixture, the relative deviation has increased, in series A, to nearly 0.55; in

B to nearly 0.28. With yet further coarsening of the mixture the deviations decrease. Note, however, that with only one exception the deviations are much less in series B ( $d_c/d_f = 41$ ) than in series A ( $d_c/d_f = 11$ ).

Provided that mixing and packing are complete, the differences between observed and theoretical values may be attributed entirely to the magnitude of the ratio of the effective diameters of the two components. The diameter ratio in both series is too small for the boundary irregularities to be without noticeable influence on the packing pattern. As the diameter ratio increases, however, the influence of one size class upon the packing pattern of the other decreases.

Consider the changing conditions in either series as the absolute minimum void ratio for the binary is approached through the fine-particle matrix: (a)

the coarse particles increase in number at the expense of the fine; (b) the number of contact areas, coarse/fine, becomes proportionately greater; and (c) the more noticeable becomes the interference with the packing pattern characteristic of the fine-particle matrix. The relative deviations, therefore, increase.

If  $d_c/d_f$  were very great, a second, coarse-particle matrix would be created at that critical binary composition (equation 7A and table 1) corresponding to the smallest possible void ratio. In nonideal binary and ternary systems it seems uncertain that the coexistence of more than one matrix will ever be completely attained. But, as the coarse particles increase and the fine decrease, interference with the coarse-particle packing pattern diminishes, a coarse-particle matrix eventually appears, and the relative deviation of the void ratio diminishes.

### Polydisperse Systems of Sand and Soil

For many years soil scientists and highway engineers have studied the bulk density of soils in relation to moisture content after compaction has been brought about in different ways (Bodman, 1931; Proctor, 1933; Bodman and Rubin, 1948; Vallerga, 1951; Day, 1955–1958; Li, 1956). Referring to the role of particle size distribution in soil compaction, Marshall (1959, p. 71) stated, "For a given method of compaction, the highest density is reached in soil that has a wide distribution of particles from

coarse to fine (as in the sandy loam of Fig. 18). In this case the fine particles fill up the gaps between the coarse. High density is not reached in soil made up of coarse or fine particles only." The figure reference in Marshall's monograph is to a bulk density-water content graph for a set of four soils of different textures compacted by the Proctor method (1933). The role of particle size distribution in compaction has been more closely examined in the experiments reported here. Also, attention has been given to the minimum bulk densities which commonly appear at low-moisture contents in the compaction process, as well as to the density maxima.

**Preparation of materials for compaction.** A series of eight mixtures with progressively higher sand content was prepared by the incorporation of sieved sand with previously sieved, air-dry <2 mm Stockton silty clay soil of known particle size distribution (table 2). The sand and soils were first dry-mixed in the dry-weight proportions, sand/silty clay: 0/100, 20/80, 40/60, 60/40, 80/20, 87/13, 93/7, 100/0. The first five (0/100 through 80/20), respectively, possess the same particle size proportions by weight as are required (Soil Survey Staff, U.S. Department of Agriculture, 1951) of: silty clay, clay loam, sandy clay loam, sandy loam, and loamy sand. The last three, containing 87 per cent or more sand, belong in the "sands" group. Where necessary, weight proportions have been converted to volume proportions from the appropriate

TABLE 2  
PARTICLE SIZE DISTRIBUTION OF STOCKTON SILTY CLAY\*

Dispersant used	U. S. sand					U. S. silt			U. S. clay	Total
	2-1 mm	1-0.5 mm	0.5-0.25 mm	0.25-0.10 mm	0.10-0.05 mm	50-20 μ	20-5 μ	5-2 μ	2 > μ	
None.....	per cent 0.08	per cent 0.33	per cent 0.67	per cent 1.64	per cent 6.90	per cent 21.2	per cent 37.6	per cent 13.7	per cent 18.9	per cent 101
H <sub>2</sub> O <sub>2</sub> + calgon	0.05	0.80	1.77	3.36	7.20	13.7	22.3	5.6	47.2	102

\* Carbon content: 2.06%; pH (water-saturated soil paste) : 7.7.



TABLE 3  
POLYNOMIAL CONSTANTS OF EMPIRICAL EQUATION [14] RELATING  
COMPACTED BULK DENSITY TO WATER CONTENT FOR  
FINE SAND-SILTY CLAY MIXTURES

Solids volume ratio, fine sand/silty clay	Constants			
	$a_0$	$a_1$	$a_2$	$a_3$
0/100 .....	2.40	$- 18.1 \times 10^{-2}$	$8.53 \times 10^{-3}$	$- 1.20 \times 10^{-4}$
20.4/79.6 .....	1.92	$- 13.4 \times 10^{-2}$	$7.85 \times 10^{-3}$	$- 1.25 \times 10^{-4}$
40.6/59.4 .....	2.04	$- 21.9 \times 10^{-2}$	$18.7 \times 10^{-3}$	$- 4.33 \times 10^{-4}$
60.6/39.4 .....	2.02	$- 25.7 \times 10^{-2}$	$29.2 \times 10^{-3}$	$- 8.79 \times 10^{-4}$
80.3/19.7 .....	1.61	$- 8.46 \times 10^{-2}$	$16.9 \times 10^{-3}$	$- 6.89 \times 10^{-4}$
85.7/13.3 .....	1.50	$- 0.344 \times 10^{-2}$	$7.5 \times 10^{-3}$	$- 4.38 \times 10^{-4}$
93.2/6.8 .....	1.56	$+ 1.16 \times 10^{-2}$	$1.77 \times 10^{-3}$	$- 1.45 \times 10^{-4}$

solid densities. The volume proportions are given in table 3 and later.

Two grades of sieved "Monterey crystal amber quartz" were used separately:  $1 < 1.19$  mm (= U.S.D.A. very coarse sand, or International coarse sand) and  $0.25 < 0.297$  mm (= U.S.D.A. medium sand, or International coarse sand).<sup>5</sup> Thus two particle size sequences were prepared, the members differing only in the sand grain-size distribution and in the class names (Soil Survey Staff, U.S. Department of Agriculture, 1951) assigned to those mixtures having more than 85 per cent sand. According to U.S.D.A. terminology, mixtures containing  $1 < 1.19$  mm sand would be termed "coarse sands" and those containing  $0.25 < 0.297$  mm sand would be named "sands." The two groups will be distinguished in this discussion by the names "coarse sand" and "fine sand" mixtures, respectively, to emphasize the almost fourfold difference in grain size of the added sands.

The sixteen dry mixtures were next given identical treatment. They were water-saturated, drained in contact with dry sand to remove excess water, further mixed and thoroughly puddled in a Hobart food mixer, removed, spread as a 1/2-in. layer on a plastic sheet, sliced into 2-in. squares for easy breakage, and allowed to dry. They were then exposed

to three additional wetting-drying cycles by gently spraying with an atomizer to about their field capacity, and air-drying three times. After breakage, the dry lumps were rubber-pestled to 1/4-in. size, further reduced by passage through a mill with widely spaced grinding plates, sieved finer than 2 mm, and stored for use.

The whole procedure was designed to prepare mixtures having some resemblance to a set of natural soils in a texture sequence in which, for each member, the chemical composition and particle shape of clay, silt, and sand were constant.

The following procedure was adopted to insure uniform wetting of the soil material before compaction for the bulk density-water content measurements. About 1 kg of the given air-dry mixture was divided into 100 gm lots and filled, layer by layer, into a glass jar, each layer being wetted when placed in the jar by adding one-tenth of the amount of water needed to bring the entire sample to the required water content. The jar was sealed and, at intervals of 24 hours, the contents were emptied on a plastic sheet and slowly hand-mixed, without puddling, then returned to the jar and resealed. After three such treatments the mixture to be compacted was placed in a covered bowl. Sufficient material

<sup>5</sup> Both size classes of sand were examined under the microscope and found to contain angular and subangular shapes. The coarse sand was more uniformly subangular than the fine sand. Despite long continued sieving the fine sand particles possessed more size variability than the coarse.

was removed, by random spoon sampling, to a copper compaction cylinder which was fitted at the top with an open, removable extension ring to avoid end effects at top of compacted sample in the lower cylinder.

**Compaction procedure.** During the filling one-sixth of the total material was placed in the cylinder, with extension ring in place, and compacted. The process was repeated with a second sixth, and so on, until all had been compacted. The topmost part of the compacted column was now cut off, horizontally, below the extension ring. Trials showed that this excess contained an amount of water representative of the moisture content of the whole column. This percentage was used in considering the final water content-bulk density relationships. The cylinder proper, with compacted contents, was then weighed, and after proper corrections the dry bulk density of the compacted material was calculated.

**Kneading compactor.** A mechanical kneading compactor, operated by compressed air and delivering pressure to the soil surface through an aluminum tamping foot, was used as an arbitrary means of compacting the soil. During compaction the soil was held in the copper cylinder, 77.5 mm high by 73.5 mm i.d. (inside diameter) and having wall thickness of 3 mm. The machine, designed by P. R. Day, has the essential function of that described by Vallerga (1951) for higher pressures. The compacting foot, 38.1 mm diameter by 12.7 mm thick, is of equilateral cross-section with sides having the same curvature as the inner wall of the cylinder and a length equal to one-sixth of the inner

wall. The tamping pressure and dwell time of the foot on soil surface are controlled by a pressure switch and a time-delay relay. When the downward-moving foot reaches the soil surface, the bearing pressure increases from zero to the prescribed maximum, the foot dwells briefly, moves upward, and so allows a return to zero pressure. A synchronized gear then rotates the cylinder so that the next downward stroke brings the foot to bear on an adjacent area of soil, and the process is repeated. There is slight overlap at the rear convex edge of the foot. Six such processes correspond to one revolution of the cylinder. Day (1955-1958), using a hand kneading compactor, has shown for several soils that a maximum pressure of 20 psi (pounds per square inch) delivered in this way, 30 strokes per layer of soil, creates a maximum bulk density similar to that in compacted zones in cultivated field soils. Preliminary experiments of our own with the experimental soils and the machine kneading compactor indicated that, on a bulk density-number-of-strokes plot ( $< 96$  strokes), 48 strokes per layer, increasing to a maximum of 20 psi, produced bulk densities lying within 10 per cent of the apparent maximum for the silty clay soil and within 1 per cent of this maximum for a sandy soil containing 80 per cent added fine sand. Accordingly, during the cylinder-filling process described earlier, the soils were compacted by a total of 48 strokes per layer, each stroke increasing to a maximum of 20 psi. Replicate specimens, when prepared and compacted with this machine in the described manner, give closely agreeing bulk densities.

## RESULTS AND DISCUSSION

In real polydisperse systems of soils having very many size classes, boundary effects presumably are pronounced, owing to the number and proportions of classes commonly involved, and to

forces of adsorption and mutual attraction displayed by the clay and organic matter.

These complexities may be simplified to some extent by thinking of the poly-

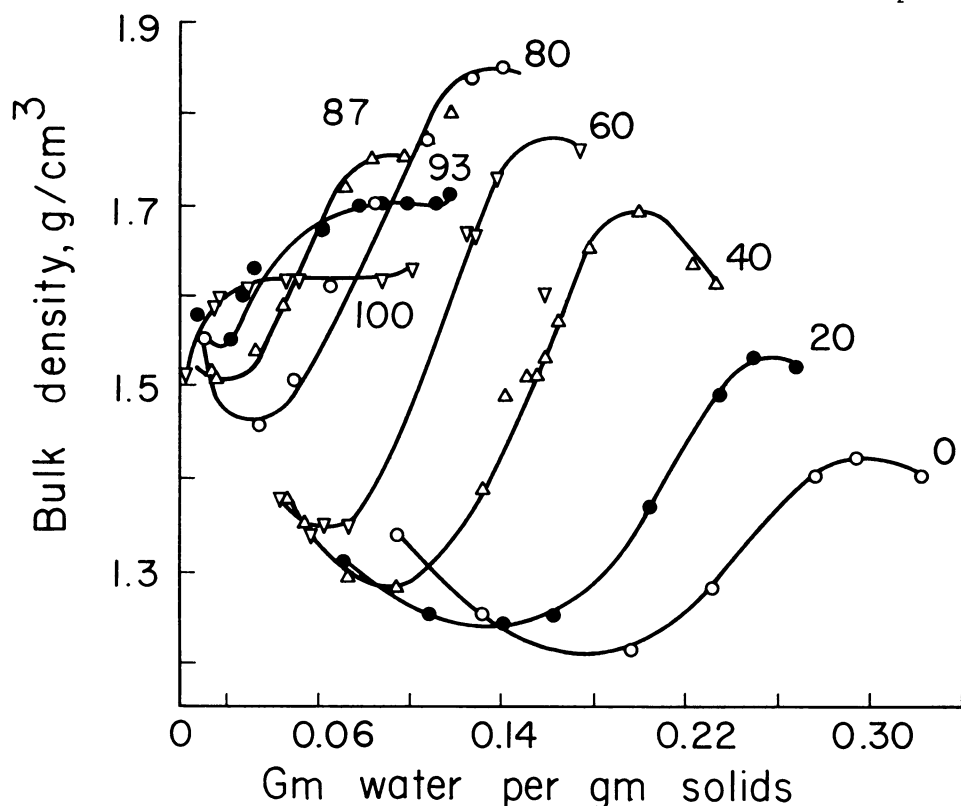


Fig. 3. Compaction curves of fine sand-silty clay mixtures. Labels on curves show percentages of added sand.

disperse system as having a greatly reduced number of size classes, each class being polydisperse but, insofar as packing is concerned, acting as a single size class in accordance with the basic assumptions described earlier. This point of view was adopted in the present experiments for the purpose of comparing observed and "theoretical" values: the added sand in the sand-silty clay mixtures was thought of as the coarse component, and the Stockton silty clay as the fine component. From this point of view, therefore, in order to examine the effect of variation in particle-size distribution upon compacted bulk densities, the compacted density and void ratio of the silty clay loam alone are required, in addition to those of the added sand. The mixtures of intermediate composition are then compacted and, with the end members, are re-

garded as a series of binary mixtures.

**Compaction curves.** The bulk density-water content functions for each composition parameter appear in figure 3. Each curve, except that for 100 per cent fine sand, shows that with increase in water content during standard compaction there is first a decrease in bulk density to a minimum, then an increase to a maximum and finally, near saturation, a second decrease. The minimum and maximum densities increased as sand content increased to 80 per cent whereas, with yet further sand increases, the minima increased and the maxima diminished. Similar curves representing higher densities were obtained for the coarse sand and soil mixtures.

For the fine sand mixtures having a matrix of silty clay, the dry bulk density ( $y$ ) - water content ( $x = \frac{\text{gm water}}{\text{gm solids}}$ ) relations are approximately represented



by equations of the following types:

for maximum bulk densities,

$$\frac{1}{y} = ax + b \quad [12]$$

for minimum bulk densities,

$$y = ke^{cx} \quad [13]$$

The particle densities of the sand and silty clay in our mixtures were unequal, so the numerical value of  $b$  in equation [12] and of  $k$  and  $c$  in [13] vary slightly from one mixture to another. They are true constants only if the mean soil particle density is the same for both components. Equation [12] gives the

bulk density of a completely water-saturated soil. As shown later (table 4), none of the mixtures were completely saturated at their maximum density.

For bulk densities and water contents lying between the maximum and minimum densities, the experimental data fit closely to cubic polynomials,

$$y = a_0 + a_1x + a_2x^2 + a_3x^3 \quad [14]$$

The remaining symbols in the equations have the meanings:

- $a$  = specific volume of water
- $b$  = specific particle volume of solids
- $e$  = base of natural logarithms
- $a_0, a_1, a_2, a_3$ , are constants.

Table 3 summarizes the equation [14] constants obtained for the fine sand mixtures.

For a given soil, the existence of a soil bulk-density maximum in the compaction curve relating its bulk density and water content, as demonstrated here, is well known. These experiments further establish the existence of a minimum bulk density throughout a series of textures. Also, it now seems that for soils of different particle size distribution the arrangement of the curves follows a definite texture sequence and that, as more clearly seen in figure 4, the position of each bulk density maximum (bulk volume minimum) occupies a rational place on a binary composition diagram based on certain gross but definable packing rules. The greatest bulk density maxima were obtained with mixtures having loamy sand textures, regardless of the particle size of the sand component.

Bulk density minima are pronounced in figure 3, and also fall into a texture sequence. Since they correspond to bulk volume maxima, however, they do not follow the same packing rules and

are not amenable to the same treatment as the density maxima. Table 4 gives the critical bulk densities obtained for the fine sand-silty clay mixtures, together with certain other associated properties.

Influence of the diameter ratio on closeness of particle packing has been shown in figure 4 by plotting the bulk volume minima obtained by separate compaction of both coarse sand-silty clay mixtures and the fine sand-silty clay mixtures. The added coarse sand, as stated earlier, has an effective diameter some four times as great as that of the fine sand. Deviations of the observed minimum bulk volumes from the corresponding theoretical minima may be seen by comparing the curves and the straight lines. To avoid confusion in figure 4, the intersecting straight lines (loci of theoretical values) for only the fine sand-silty clay mixtures have been drawn since, in the silty clay matrix, the theoretical values for the coarse sand mixtures would have been superimposed on those for the fine sand mixtures and would have been very close to the latter in the sand matrix region. The plot of the bulk volume minima of the coarse sand-silty clay mixtures lies noticeably closer to the theoretical values than do those of the fine sand mixtures. This is attributed mainly to the increased diameter ratio in the

TABLE 4  
PROPERTIES OF BINARY MIXTURES OF FINE SAND AND SILTY CLAY SOIL  
AT THEIR MINIMUM AND MAXIMUM BULK DENSITIES

Solids volume ratio, fine sand/silty clay	Mean particle density  <i>gm/cm<sup>3</sup></i>	At minimum bulk density					At maximum bulk density				
		Bulk density <i>gm/cm<sup>3</sup></i>	Void ratio	Suction  <i>mb</i>	Water content  <i>per cent</i>	Pore saturation*   	Bulk density† <i>gm/cm<sup>3</sup></i>	Void ratio	Suction†  <i>mb</i>	Water content  <i>per cent</i>	Pore saturation*   
0/100 . . . . .	2.710	1.21	1.240	> 975	18.7	0.409	1.42	0.908	537	30.0	0.895
20.4/79.6 . . . . .	2.696	1.24	1.174	> 975	13.4	0.307	1.53	0.762	480	25.6	0.906
40.6/59.4 . . . . .	2.682	1.28	1.095	> 975	9.0	0.220	1.69	0.585	226	20.0	0.916
60.6/39.4 . . . . .	2.668	1.34	0.991	> 975	6.4	0.173	1.77	0.507	415	16.3	0.858
80.3/19.7 . . . . .	2.654	1.46	0.818	> 975	3.2	0.104	1.85	0.434	47	13.8	0.843
86.7/13.3 . . . . .	2.650	1.51	0.755	> 975	2.2	0.075	1.75	0.514	200	9.2	0.475
93.2/6.8 . . . . .	2.644	1.54	0.717	> 975	1.6	0.059	1.70	0.555	36	10.0	0.476
100/0 . . . . .	2.640	1.49	0.772	> 975	0.4	0.014	1.62	0.629	34-38	5-9	0.281

\* Pore saturation =  $\frac{\rho_w w}{\rho_{sw}}$ , in which  $\rho_s$  = particle density  $\rho_w$  = water density  $w$  = water content as percentage by wt., dry basis/100  
† The theoretical greatest maximum bulk density, 2.17 gm/cm<sup>3</sup>, occurs at the composition 75.2 per cent fine sand and 24.8 per cent silty clay.  
‡ Suctions obtained by linear interpolation from desorption measurements at 21°C (see figures 5 and 6). At this temperature 1 millibar (mb) = 1.022 cm water.

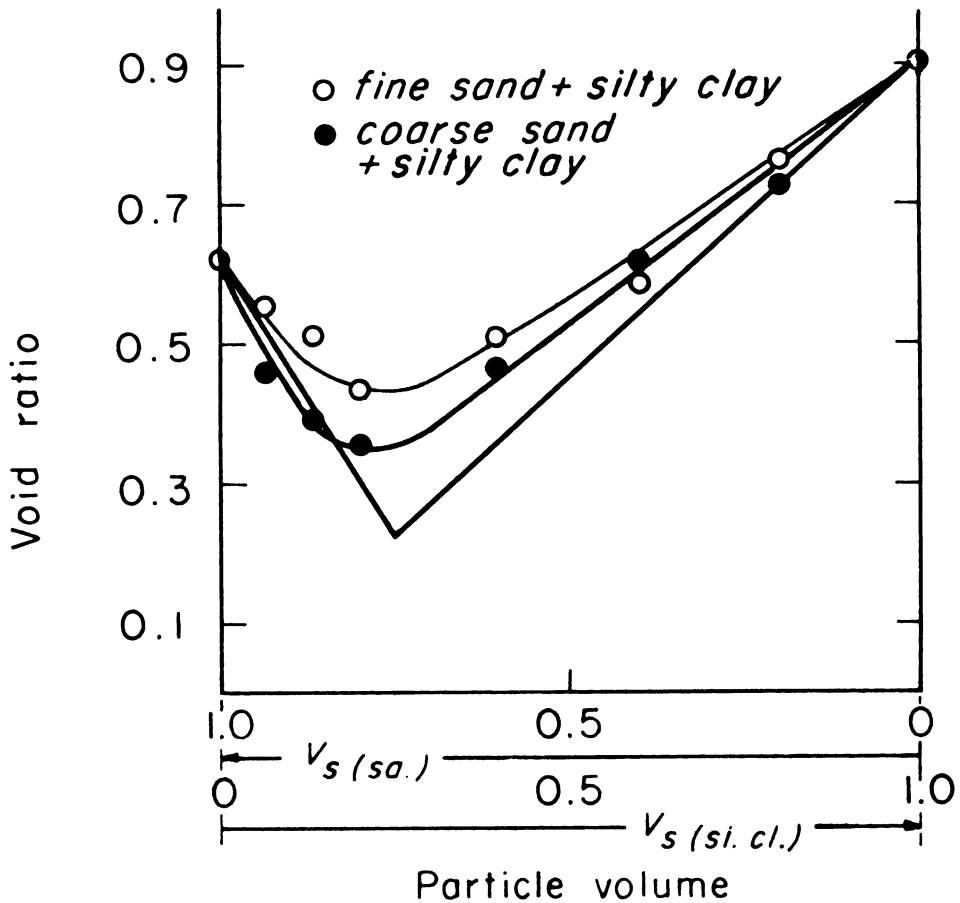


Fig. 4. Minimum void ratios of binary mixtures of sand and silty clay in relation to composition, observed and theoretical, showing effect of diameter ratio.

coarse sand mixtures as well as to frictional resistance to bulk volume reduction in the fine sand mixtures, to some slight interaction at fine sand particle surfaces, and possibly to entrapped air. Mechanical interference is inevitable in any practical method of packing real soil particles more closely and in field compaction. Departures from theoretical behavior, however, generally must be attributed also to nonideal particle properties.

The theoretical minimum void ratios for the fine sand mixtures of figure 4, including the smallest minimum, and the theoretical minimum bulk volumes, together with their binary composition values may, of course, be obtained directly by carefully reading the graph.

They may also be obtained by use of the equations for the fine sand and silty clay matrices, respectively. The added fine sand is much coarser than the silty clay so that, for the present purpose, the subscript  $c$  may be used for the fine sand and the subscript  $f$  for the silty clay. For the fine sand matrix,  $V_{bcf} = V_{sc} (1 + e_c)$ ; for silty clay matrix,  $V_{bcf} = V_{sf} (1 + e_f) + V_{sc}$ . At the lowest minimum the two straight lines corresponding to these equations intersect so that  $V_{sc} (1 + e_c) = V_{sf} (1 + e_f) + V_{sc}$ , and we obtain  $V_{sf}/V_{sc} = e_c / (1 + e_f)$ . (This is equation [7A].) From the appropriate measured void ratios (table 4, lines 1 and 8 under column 9,  $e_f = 0.908$ , and  $e_c = 0.629$ ) and from equations [1] and [7A], we obtain  $V_{sf} = 0.248$  and  $V_{sc} =$



0.752 at the smallest minimum bulk volume. The mean particle density for these proportions is  $2.658 \text{ gm/cm}^3$  (table 4, by interpolation in column 2). From the smallest minimum bulk volume of the mixture (either sand matrix or clay matrix equation above) and the mean particle density, we obtain: theoretical smallest minimum void ratio = 0.225, corresponding minimum bulk volume =  $(1 + 0.225) = 1.225$  units, and greatest maximum bulk density =  $2.658/1.225 = 2.17 \text{ gm/cm}^3$ . It is seen that the lowest experimental void ratio greatly exceeds 0.225 (table 4), but occurs at a volume composition ratio (table 4 and figure 4)  $V_{s(\text{fi.sa.})} / V_{s(\text{sl.cl.})}$  that is very close to the theoretical composition for such minimum.

**Effect of incorporated polymer on compaction.** An additional set of fine sand-silty clay mixtures was compacted after prior incorporation, in the dry state, of 0.4 per cent by weight of vinyl acetate maleic anhydride (VAMA). In all other respects the precompaction treatments of these soils were identical with the previously described treatments given the same soils for which VAMA was absent. The results for maximum and minimum bulk densities, with VAMA, are contained in table 5.

The compaction curves were similar to those obtained earlier. The bulk density minima, as before, were found at the lower moisture contents, and the magnitudes of the minima diminished with increasing abundance of fine particles. Compared to soils lacking VAMA, the minima appeared to be only slightly lowered by the added VAMA, and this only with soil mixtures containing 80 per cent or more added fine sand. The minima occurred, however, at lower water contents and at lower pore saturations. The maximum densities, on the other hand, were significantly diminished by VAMA additions to all but the pure fine sand, which was unaffected. An average decrease of  $0.18 \text{ gm/cu. cm}$  in maximum bulk density was obtained with the silty clay and the six mixtures,

the most pronounced decreases (about  $0.25 \text{ gm/cu. cm}$ ) occurring in and near the middle part of the binary sequence. For all mixtures except that having the greatest maximum bulk density, the reduction produced by VAMA was accompanied by an increased water content at which the maximum was produced. Within the silty clay matrix group, there was also a decrease in pore saturation. Mixtures in the sand matrix showed an increase in degree of saturation.

**Desorption.** Desorption curves were obtained for three lots of compacted fine sand-silty clay mixtures by measuring suction and water-content values at suctions of from 10 to 1000 cm of water, and also the water content at zero suction, toward which complete saturation was aimed and which was assumed to exist. The mixtures were first brought to water contents previously found necessary to produce the maximum and minimum densities for each mixture. Two of the three lots were compacted to their minimum and maximum densities before desorption. The third lot was first mixed with 0.4 per cent by weight of VAMA, followed by compaction of each mixture to the maximum density before desorption. Precompaction procedure for all materials was exactly the same as that adopted and prescribed for the other experimental runs.

Attempts were made, first, to transfer sections of the compacted soils from copper cylinder to desorption plate. Owing to difficulty in avoiding irregular breakage and loss with the noncoherent, low-density members during transfer, however, these efforts were abandoned. A special small cylinder and porous plate were constructed, as a unit, in which a shallow column of soil could be placed for retention during standard kneading compaction directly against the plate. Transfer was thus avoided and desorption could begin as soon as suitable connections enabled pressure differences to be established across the plate.

TABLE 5

EFFECT OF INCORPORATED VINYL ACETATE MALEIC ANHYDRIDE (VAMA) UPON MINIMUM AND MAXIMUM BULK DENSITIES AND ASSOCIATED PROPERTIES OF FINE SAND-SILTY CLAY MIXTURES

Properties at critical densities	fine sand (per cent)					
	Solids volume ratio,			silty clay (per cent)		
	0	20.4	40.6	60.6	80.3	93.2
	100	79.6	59.4	39.4	19.7	6.8
						100
						0
At minimum bulk density: VAMA absent						
Minimum bulk density (gm/cm <sup>3</sup> )	1.21	1.24	1.28	1.34	1.46	1.51
Void ratio	1.24	1.17	1.1	0.991	0.818	0.755
Water content (wt. per cent)	18.7	13.4	9.0	6.4	3.2	2.2
Pore saturation	0.409	0.307	0.22	0.173	0.104	0.075
						0.059
						0.014
At minimum bulk density: VAMA present						
Minimum bulk density (gm/cm <sup>3</sup> )	1.21	1.27	1.27	1.3	1.34	1.4
Void ratio	1.24	1.13	1.11	1.05	1.12	0.9
Water content (wt. per cent)	16.3	11.0	8.2	4.8	1.1	0.8
Pore saturation	0.356	0.262	0.198	0.122	0.026	0.024
						0.013
						0.01
						1.49
						0.772
						0.3
						0.01
At maximum bulk density: VAMA absent						
Maximum bulk density (gm/cm <sup>3</sup> )	1.42	1.53	1.69	1.77	1.85	1.75
Void ratio	0.908	0.762	0.585	0.507	0.434	0.514
Water content (wt. per cent)	30.0	25.6	20.0	16.3	13.8	9.2
Pore saturation	0.895	0.906	0.916	0.858	0.843	0.475
						0.476
						1.62
						0.029
						5<9
						0.281
At maximum bulk density: VAMA present						
Maximum bulk density (gm/cm <sup>3</sup> )	1.32	1.39	1.45	1.49	1.6	1.61
Void ratio	1.05	0.94	0.85	0.791	0.664	0.649
Water content (wt. per cent)	32.4	27.2	23.6	22.0	13.8	12.4
Pore saturation	0.836	0.78	0.745	0.742	0.552	0.506
						0.619
						1.64
						0.61
						8.4
						0.364

The cylinder consisted of two concentric rings of epoxy plastic, each 15 mm high by 80 mm i.d., and rested directly upon a cylindrical bottom section of equal o.d. (outside diameter). A porous alundum plate,<sup>6</sup> having a ribbed and grooved lower surface, was fitted into a countersunk recess in the upper part of the bottom section, forming the bottom for the two upper rings. A chamber 2 to 3 mm deep accommodated, and its bottom surface supported, the lower part of the plate, allowing the escape of water through an external nipple. A small vent, fitted with a plug for sealing, was provided to remove any air entrapped in the chamber.

Before kneading compaction the two rings and the bottom porous-plate section were clamped together. Sufficient soil was added from the moistened stock supply to fill the lower ring and was compacted, in position, with 48 strokes at 20 psi. A second soil layer was then added to fill the remaining space in the cylinder and similarly compacted. The clamps were next removed and the upper ring separated from the lower. The soil column was cut so as to leave a lower 15 mm-high segment of compacted soil surrounded by the lower ring and in contact with the alundum plate. To minimize volume changes in the upper few millimeters of the mixtures during water uptake, a surface-bearing pressure of about 1 psi was applied by placing a fine metal wire screen and weighted plate on top of the column. A plastic tube,  $\frac{1}{8}$  in. o.d., was attached to the nipple, the soil and the porous plate were saturated under vacuum with air-free water, and the entire assembly was then placed in a gas pressure chamber without loss of water. Connection was made from the plate chamber to the bottom of a vertical burette. Care was taken during the transfer and connection process to keep the connecting tube opening at the soil level to avoid subjecting soil water to pressure changes.

A series of pressure increases was now

applied to the moist soil so as to produce a total of nine known suctions, from 10 to 1000 cm water. Equilibrium was assumed to have been reached, under any given suction, if water outflow ceased for 2 hours. The mean laboratory temperature during desorption was about 21°C with only small fluctuations.

The volume of water removed in relation to applied suction forces was measured by inspection of the meniscus level in the burette. This level was always maintained equal, within 1 mm, to that of the soil sample by an electric relay and motor geared to the burette support. With progressive desorption the rising meniscus was made to close an electric circuit, causing the motor to lower the burette 1 mm. The instant of each lowering was punched on recording tape, which provided the criterion for equilibrium.

After equilibration with a suction of 1000 cm of water (979 millibars at 21°C) the burette stopcock was closed to prevent water re-entry, the pressure released, and the pressure cylinder opened. The soil cylinder was moved sideways off the porous plate and weighed immediately. The water remaining in the soil against this suction was determined by drying to 105°C. Amounts of water released and retained were calculated in terms of volume-per-unit volume of solids and their sum was assumed equal to the initial void ratio of the saturated, compacted soil.

Desorption curves for the fine sand-silty clay mixtures are shown in figures 5, 6, and 7 which refer, respectively, to soil mixtures first compacted to their minimum bulk densities (5), the same mixtures first compacted to their maximum bulk densities (6), and to mixtures with which VAMA had been incorporated before compaction to maximum densities (7). The water content is expressed in these figures as cm<sup>3</sup> water/cm<sup>3</sup> solids. It may be converted to a weight ratio, dry basis, by dividing by the specific gravity of the corres-

<sup>6</sup> Alumina powder, A.D. 85 (Coors), was fired for 1 hour at 1750°F to make the plate.

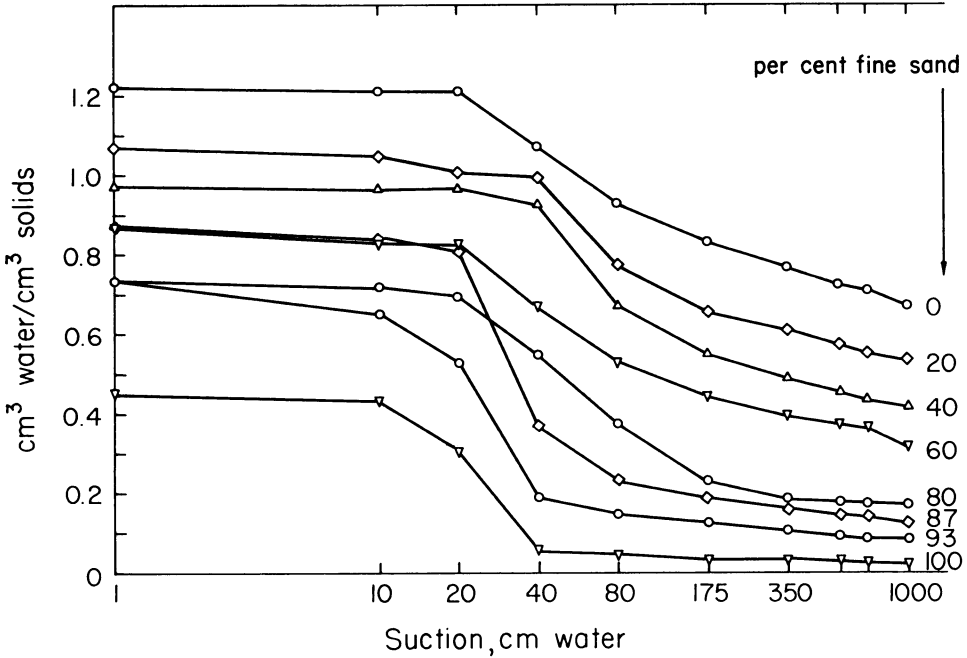


Fig. 5. Suction desorption curves of fine sand-silty clay mixtures after compaction to minimum bulk density.

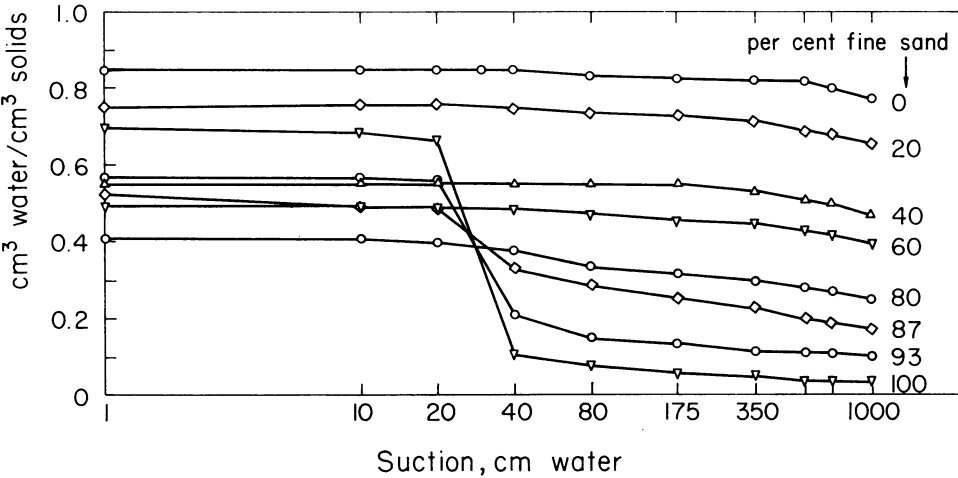


Fig. 6. Suction desorption curves of fine sand-silty clay mixtures after compaction to maximum bulk density.



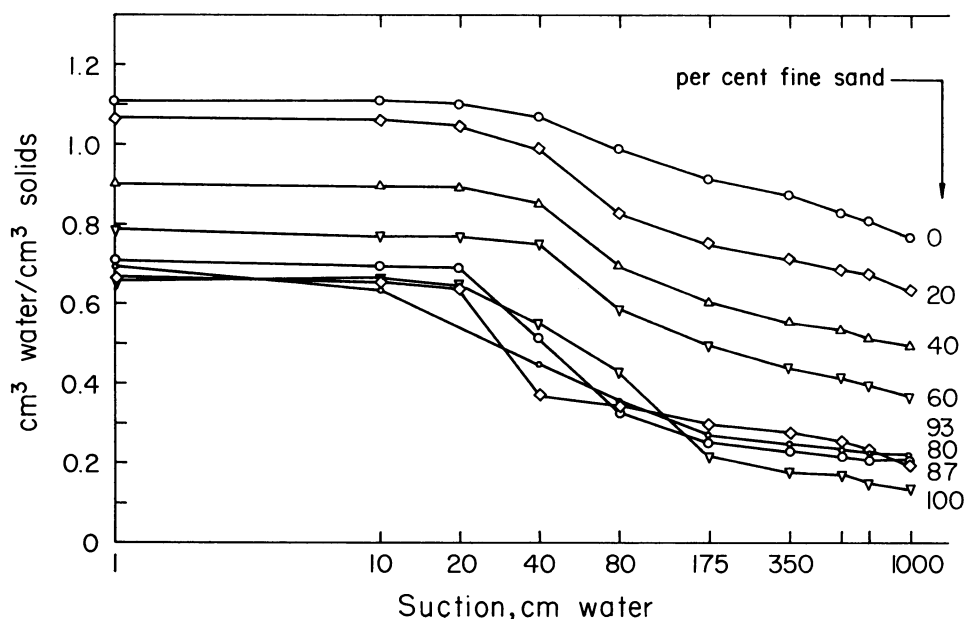


Fig. 7. Suction desorption curves of fine sand-silty clay mixtures after VAMA incorporation and compaction to maximum bulk density.

ponding mixture (see table 4 for mean particle densities). Since the specific gravities of all mixtures are similar, the mean specific gravity of all may be used, and a close approximation to the percentage of water by weight, dry basis, is then obtainable for any mixture by multiplying the volume ratio by 37.5 ( $= 100/2.668$ ).

Table 4 presents properties of the binary mixtures at their critical densities. All entries are equilibrium values, the suctions having been measured, as described, by compaction of separate samples on a porous plate after first being brought to the moisture contents previously established for their critical densities. Except for very small evaporation loss around the tamper, no water was lost from samples during compaction, and  $V_w / V_s$  remained substantially constant. Compaction, however, reduces the size and total volume of pores, so that equilibrium suctions after compaction are not the same as before.

The desorption curves for the mix-

tures at their minimum densities (figure 5), beginning at saturation, are characterized by sharp breaks at low suctions: 10 cm for the two mixtures with the most sand, and about 20 cm for the remainder. A texture influence seems clear over the measured range of suctions; the 86.7 sand/13.3 silty clay mixture is an exception. At suctions less than 20 cm this mixture holds amounts of water equal to those of the 60.6 sand/39.4 silty clay mixture. At higher suctions, however, the curve for the former mixture shifts and assumes a consistent position in the texture sequence. Desorption at a suction of 979 millibars (1000 cm water suction) reduced the water content of all mixtures at their *minimum* bulk densities below that to which the corresponding mixtures were reduced at this suction after compaction to their *maximum* densities (cf. figure 6).

The void ratios of the mixtures at their maximum bulk densities lie between 50 and 80 per cent of those at the

minimum densities. At full saturation, therefore, and at zero tension, the volume of water retained per unit volume of solids at maximum density is proportionately less. The desorption curves indicate this difference (figure 6). Moreover, no sharp breaks appear in the curves for any of the mixtures containing 80 per cent or less of added fine sand, although the rate of loss with increasing suction is slightly greater at suctions exceeding 40 to 80 cm than it is at the very low suctions. The curves for mixtures having 87 per cent and more of added fine sand break sharply at suctions of 20 cm. Evidently these two categories of desorption curves for the maximum densities belong, respectively, to the silty clay and sand matrices. The curve for the 80 per cent fine sand:20 per cent silty clay mixture is of intermediate form. This mixture has a sand matrix but contains only slightly more sand than is necessary to cause transformation from the matrix of silty clay. The entire set of desorption curves thus appears to reflect clearly the differences in particle size composition as these differences are associated with the alterations in packing induced by compaction.

The incorporation of 0.4 per cent VAMA before compaction to the maximum bulk density does much to preserve an open packing of the soil particles. The desorption curves of figure 7 indicate that, although the mixtures were previously brought to the water contents necessary for compaction to their maximum bulk densities and then compacted, the effective particle arrangement—as shown by water release under suction—is as though compaction had been brought about at much drier conditions. The curves are generally very similar in position and shape to the desorption curves obtained for the minimum bulk densities. At suctions greater than 20 cm water, however, the fine sand alone with which VAMA has been mixed retains, after compaction to

its maximum bulk density, even more water per unit volume of solids than it retains after compaction to its minimum density in the absence of VAMA. Similar effects are observed over much of the suction range for the 93.2/6.8 cm, and 86.7/13.3 mixtures. The effect of VAMA on water retained at 1000 cm of water suction is relatively small, except for the soils having a sand-matrix packing.

**Mean effective pore size.** The capillary rise equation was used for formal calculation of the mean effective pore diameter corresponding to each equilibrium suction applied to the soil. Upon this basis the volumes of pores having different mean effective diameter limits were calculated for each compacted sample by assuming no soil shrinkage and a water density, before withdrawal, of 1 gm/cu. cm for all suctions up to 979 millibars (table 6). Figure 8 includes curves indicating pore-size frequency for all of the compacted

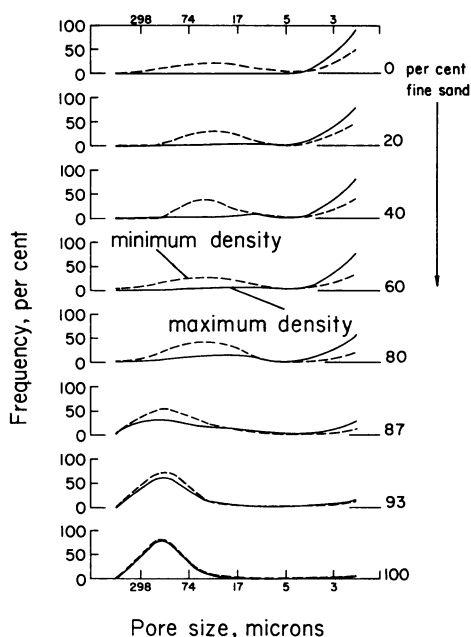


Fig. 8. Frequency distributions of mean effective pore diameter after compaction of mixtures to their minimum and maximum bulk densities.

TABLE 6  
DISTRIBUTION OF CALCULATED MEAN EFFECTIVE PORE DIAMETERS AT MINIMUM AND MAXIMUM BULK  
DENSITIES OF FINE SAND-SILTY CLAY MIXTURES

Solids volume of added sand (per cent)	Bulk density	Water content at saturation (wt. per cent)	Mean effective pore diameter (microns)									
			> 298	298-149	149-74.5	74.5-37.3	37.3-17	17-8.5	8.5-5.4	5.4-4.3	4.3-3.0	3.0 >
			Distribution as percentage of total pore volume									
0.0	min. max.	41.3 31.2	0.8 0.0	0.0 0.0	11.2 0.0	11.6 0.6	8.0 0.6	5.6 2.0	3.5 0.5	1.0 2.5	3.4 3.6	54.9* 90.2
20.4	min. max.	39.8 30.0	2.4 1.0	3.7 0.6	0.8 0.4	21.2 1.0	10.7 1.5	4.5 1.2	3.0 3.6	1.8 1.0	2.0 9.0	49.9 80.7
40.6	min. max.	36.2 20.7	0.8 0.5	0.0 0.7	4.2 0.5	25.7 0.2	13.1 0.2	6.6 3.1	3.2 4.9	1.6 0.7	2.0 5.4	42.8 83.8
60.6	min. max.	32.3 18.6	4.1 1.4	0.6 0.2	18.1 1.0	16.0 2.6	10.1 3.0	5.1 2.6	3.1 3.2	0.6 1.8	5.7 4.6	36.6 79.6
80.3	min max.	27.6 15.4	1.9 1.5	3.1 1.0	20.1 5.9	23.5 9.8	19.7 4.2	6.7 4.9	0.5 4.4	0.4 2.9	0.5 5.4	23.6 60.0
86.7	min. max.	33.1 19.8	3.8 5.5	4.1 2.5	50.2 29.3	14.8 9.3	6.2 5.7	2.5 5.5	2.2 4.6	0.1 2.3	2.2 3.6	13.9 31.7
93.2	min. max.	28.0 21.5	1.5 1.2	2.7 1.8	69.9 60.6	6.3 9.7	3.4 3.5	2.6 3.3	1.9 1.1	0.3 0.4	0.6 1.1	10.8 17.3
100	min. max.	16.9 26.6	2.9 2.7	28.5 2.7	55.6 79.7	3.1 3.4	2.0 3.1	0.0 1.3	1.4 1.9	1.1 0.1	0.4 0.0	5.0 5.1

\*Percentage of pores  $3\mu$  m.e.d. > obtained by difference.

mixtures, VAMA absent. Frequency curves for the VAMA-treated, maximally compacted mixtures are omitted so as to avoid confusion with those for the untreated minima to which they approximate and which they would, in large part, overlap.

To smooth the experimental data for plotting and aid comparison of pore-size frequency in the mixtures at both critical densities, the eight pore-size classes between  $298\ \mu$  and  $3\ \mu$  were reduced to four by combining the two volumes for each pair of adjacent size classes into one volume for a single size class of increased range. All size classes are represented by an equal distance along the abscissa for the same reason, and a smooth curve has been drawn through the appropriate points of the classes for each degree of soil compaction. Points for the minimum and maximum densities, respectively, have been shown by broken and continuous lines.

The mean effective pore diameter (m.e.d.), if calculated from an energy expression for soil water at very low energy levels, generally is meaningless, even though any necessary corrections are made for dissolved substances. When calculated, as in the present instance, from relatively low suctions ( $< 1000$  millibars) the m.e.d. is less subject to criticism. The purpose here is simply to indicate something of the interior pore organization so that comparisons may be made between the two extreme states of compaction of a given mixture.

It is noticeable that, within the range of measured suctions, different frequency distributions of pore m.e.d. exist between mixtures that have been compacted to their minimum and maximum bulk densities, respectively. Pure fine sand possesses practically no fine pores, but, for minimally compacted mixtures and for pure silty clay, there is a bimodal frequency distribution. Compaction to maximum bulk density nearly destroys those pore size groups greater than  $5\ \mu$  m.e.d. for the silty clay and for

the two mixtures containing 20.4 and 40.6 per cent added fine sand. This category of pore size is also greatly reduced by compaction to the maximum density of the mixture containing 60.6 per cent added sand. Maximum compaction of none of the mixtures reduces the abundance of  $< 3\ \mu$  pores. Although the proportion of coarse pores is greater for the more sandy mixtures at their minimum densities, the amount of destruction of all pore size classes greater than  $3\ \mu$  m.e.d. by compaction to density maxima progressively decreases as the sand content increases beyond 60 to 80 per cent. The frequency distribution curves for pores in the pure fine sand at minimum and maximum compaction are almost identical.

As defined earlier, an ideal coarse-particle matrix is one in which all monodisperse coarse particles present touch to form a continuous "matrix" having a characteristic proportion of voids after the mixture assumes its minimum bulk volume. In this state all finer particles are to be found within the voids of the coarser particles, but in no way do they affect the packing of the coarser—that is,  $e_c$  remains unaffected. Ideally, the two components in our mixtures would form coexistent coarse and fine matrices at 75.2 per cent fine sand/24.8 per cent silty clay (see fine sand-silty clay curve, figure 4). An amount of sand in the mixture greater than 75.2 per cent by volume results in the production of a sand matrix alone at the minimum bulk volume. Note that destruction of coarse pores first appears to show marked diminution in those mixtures containing more than 80 per cent of fine sand, an amount of fine sand somewhat above the critical quantity, and that destruction has practically ceased on maximum compaction of the pure fine sand. At maximum compaction over the range 0 to 60.6 per cent fine sand, a silty clay matrix is dominant and the average loss—decrease in percentage of pores  $3\ \mu$  m.e.d.  $> /$  de-



crease in percentage of silty clay—is 0.18. This rate of loss is much smaller than can be attributed to the reduced amount of silty clay present. A possible explanation is that new pores, finer than  $3\ \mu$  m.e.d. and resistant to maximum compaction, are formed during compaction in greater numbers as more sand is added, at or near the sand grain boundaries, and the original pores finer than  $3\ \mu$  are preserved, particularly in

the protected regions where there is local “bridging” of sand grains.

Increase in content of fine sand from 80.3 to 100 per cent, with a corresponding decrease in silty clay, is accompanied by a loss of 2.8 per cent of pores  $3\ \mu$  m.e.d. and finer per unit percentage decrease in silty clay. This amount is considerably less than can be ascribed to the decreased content of silty clay, and no explanation is offered.

## CONCLUSIONS AND SUMMARY

Consideration of homogeneous, tridisperse systems, whose different particle size classes possess certain definite properties but any homogeneous particle shape and a packing behavior definable by their void ratios, leads to the development of relationships between particle size distribution and minimum bulk volume. Knowledge of the properties and void ratios of the component classes thus permits prediction of the minimum bulk volume of a homogeneous mixture that may be prepared from them.

It was anticipated that an ideal system of this kind might be used as a guide to the composition–bulk density properties of compacted soils. Trials accordingly were made with a series of sand and silty clay soil mixtures in which the sand and silty clay soil, respectively, were regarded as the coarse and fine components. These were mixed in different, known volume proportions covering a wide range of soil textures and were prepared so as to simulate real soils.

Measurements established that after compaction with a kneading compactor, maximum and minimum bulk volumes are created, at different moisture contents, for each member of the texture series except pure sand, and minimum bulk volumes are, in fact, related to an ideal pattern resulting from the proportions of different size classes present. Deviations from theoretical bulk-volume minima are explained by the narrow diameter ratios between the two

components, by interactions between particles including frictional resistance during packing, and possibly by trapped air in the compacted specimens. These interferences are all excluded in developing the ideal relationships. Deviations are measurably diminished by increasing the particle diameter ratio.

A marked change in suction-desorption curves is brought about by compaction to minimum bulk volume. The existence of fine and coarse particle “matrices” over two different particle size distribution ranges explains the positions, with respect to particle size composition, of the change in shape of the desorption curves. Changes in the frequency distribution of calculated mean effective diameter of pores are explained similarly. Maximum compaction by kneading compactor greatly increases the proportion of pores of m.e.d.  $< 3\ \mu$  at the expense of coarse pores, but the proportion of  $< 3\ \mu$  pores is associated primarily with the presence of the silty clay component. Pores of the limiting upper size of  $3\ \mu$  are practically absent in both the minimally and maximally compacted fine sand, except after the addition of 0.4 per cent VAMA. The presence of this quantity of VAMA in the mixtures had a very slight effect on their maximum bulk volumes after compaction but, under maximum compaction, caused significant increases of minimum bulk volume and so lowered the maximum bulk density.

## LITERATURE CITED

## AMERICAN GEOLOGICAL INSTITUTE

1957. Glossary of geology and related sciences. Amer. Geol. Inst., NAS-NRC, Publ. 501. Washington, D.C.: Williams and Heinz Litho. Corp.

## AMERICAN SOCIETY OF AGRICULTURAL ENGINEERS (Subcommittee of the Soil Compaction Committee, with the Soil Science Society of America)

- n. d. Annotated bibliography on soil compaction. Amer. Soc. Agr. Engin., St. Joseph, Mich. BODMAN, G.B.

1931. Observations on soil consistency and structure. Report of the Eleventh Annual Meeting, American Soil Survey Assoc. Bul. 12:93-96.

## BODMAN, G. B., and J. RUBIN

1948. Soil puddling. Soil Sci. Amer. Proc. 13:27-36.

## DALLAVALLE, J. M.

1943. Micromeritics. New York: Pitman Publ. Corp. First ed., chap. 6.

## DAY, PAUL R.

- 1955-1958. The susceptibility of soils to changes in bulk specific volume and pore size, and the effect of these changes upon related physical properties. Ann. Rept. California Contributing Project to U.S. Dept. Agr. Tech. Comm. W-30 (duplicated).

## LI, C. Y.

1956. Basic concepts on the compaction of soil. Proc. Amer. Soc. Civil Engin., Jour. Soil Mech. and Found. Div. Paper 862:1-20.

## MARSHALL, T. J.

1959. Relations between water and soil. Commonwealth Bur. Soils, Commonwealth Agr. Bur., Farnham Royal, Tech. Commun. 50:70-74.

## PROCTOR, R. R.

1933. Fundamental principles of soil compaction: Description of field and laboratory methods. Engin. News Rec. 3:286-89.

## SOIL SURVEY STAFF, U. S. Department of Agriculture, Bureau of Plant Industry and Agricultural Engineering

1951. Soil survey manual. U.S. Dept. Agr. Handbook 18:207-11, Washington, D.C.

## VALLERGA, B. A.

1951. Recent laboratory compaction studies of bituminous paving mixtures. Proc. Assoc. Asph. Pav. Technologists 20:117-53.

## WESTMAN, A. E. R., and H. R. HUGILL

1930. The packing of particles. Jour. Amer. Ceramic Soc. 13:767-79.

## WESTMAN, A. E. R.

1936. The packing of particles: Empirical equations for intermediate diameter ratios. Jour. Amer. Ceramic Soc. 19:127-29.





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