# HILGARDIA 

 A JOURNAL OF AGRICULTURAL SCIENCEPUBLISHED BYFHECALIFORNIA AGRICULTURALEXPERIMENTSTATION

Volume 36, Number 4 • December, 1964


# Size and Location Factors Affecting California's Beef Slaughtering Plants 

S. H. Logan and G. A. King



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## THE AUTHORS:

S. H. Logan is Assistant Professor of Agricultural Economics and Assistant Agricultural Economist in the Experiment Station and on the Giannini Foundation, Davis; G. A. King is Associate Professor of Agricultural Economics and Associate Agricultural Economist in the Experiment Station and on the Giannini Foundation, Davis.

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## INTRODUCTION

California ranks second among the states in number of cattle slaughtered. In 1962, California registered a total cattle slaughter of $2,565,000$ head, as against Iowa's 2,907,000 (U. S. Dept. of Agr., 1963). ${ }^{2}$

The number of slaughtering plants in California has declined over recent years; on March 1, 1960, there were 118 commercial slaughter plants in the state, but there were 131 in 1955. Despite the over-all decrease, the number of establishments slaughtering only cattle and calves increased from 22 to 26 during the same time period. Diversified plants slaughtering cattle, in addition to other species such as hogs or sheep, dropped from 109 to 91 (U. S. Agr. Mktg. Serv., 1960).

California has been a "deficit" state with regard to shipment of cattle and calves for some time. Total inshipments of cattle and calves in 1960 amounted to $1,885,000$ head, of which 450,000 were for immediate slaughter. Some 56.2 per cent of the inshipments of slaughter cattle originated in Arizona, with the rest generally scattered among other western and Great Plains states. Texas is the largest single shipper of stocker and feeder cattle to California, with

496,000 head of the total $1,435,000$ shipped in 1960 (Calif. Crop and Lvstk. Rptg. Serv., 1961).

The growth of specialized cattle slaughtering while over-all number of plants has decreased, plus the strong dependence on inshipments of live animals, makes the problems of optimum size and optimum location of slaughter plants particularly relevant to California. At present, the size and location of cattle slaughtering plants is highly varied. Daily kill rates vary from less than 25 to 450 head.

Approximately 50 per cent of the state's slaughter is centered in the Vernon area of Los Angeles County, but plants are scattered throughout the state. The present location pattern reflects decisions made over many years, and reasons for particular plant locations may no longer be valid. For example, plants were located near terminal markets receiving animal inshipments by rail. The increased use of truck transportation for live animals and for meat has contributed to the decline of central markets in California and elsewhere. Another change is the growth of concentrated feedlot areas. Thus, decisions as to new plant location would now

[^0]be based on a different set of criteria than those which were relevant when present plants were established.

The object of this analysis is to estimate the optimum number, size, and location of cattle slaughtering plants in California, and the optimum shipment patterns for live animals and carcass beef, given: (1) regional levels of live animal supply and consumer demand for meat, (2) assembly and distribution cost functions, and (3) processing costs. The level of cattle feeding in California and the inshipments of slaughter weight animals and dressed beef are, therefore, considered as given. The analysis relates to 1960 conditions, and the solution is applicable to the long-run location pattern only if supply and demand conditions were to remain
unchanged. However, if cattle feeding increases, more (or larger) plants would be justified. Conversely, if fewer slaughter weight animals are shipped into California, the solution overstates the required slaughter capacity for the state. The analysis does provide a bench mark as to plant location, size, and shipment patterns consistent with a minimum cost assembly of live animals, slaughter, and shipment of meat to consuming regions. The results should be of interest to individual firms in the slaughter industry in planning possible adjustments in size and location. In addition, the study provides a methodological approach that may be of value to research workers concerned with efficiency in marketing.

## FRAMEWORK OF ANALYSIS

## THEORETICAL CONSIDERATIONS

The location of economic activity under a competitive system depends upon product demand, supply, and transfer costs among regions. Here, attention is focused on factors affecting the location of processing plants for a single agricultural product. The framework will be that of partial equilibrium and will be concerned with raw product as well as final product transfer costs. The relevant theoretical dimensions thus are the basis for interregional trade flows, and the factors that influence location and size of processing plant facilities.

Interregional trade flows result from regional price differences that are more than the cost of shipping the product. This may be illustrated by the wellknown back-to-back diagram, in which the equilibrium trade flow and regional prices may be determined if the relevant supply and demand functions and transfer costs are given. Figure 1 shows a two-region example. The supply and
demand curves for region 2 (shown on the left half of the diagram) result in a pretrade price (OS) exceeding that in region $1(\mathrm{PQ})$ by more than transfer costs $\left(\mathrm{t}_{12}\right)$. Region 1 will therefore ship goods to region 2, thus increasing the effective supply in the latter region and decreasing the price. The diversion of some of the supply in region 1 to region 2 will simultaneously result in a higher price in region 1. Trade will continue to occur so that in equilibrium the price in the importing region equals that in the exporting region plus transfer costs, with quantity $a$ shipped from region 1 to region 2. These results may be generalized to the multimarket case (Samuelson, 1952). This formulation is that of the point-trading model in that all production and consumption occur at specified locations. The alternative continuous location model, although theoretically appealing, has not been used in this analysis due to computational restrictions. The theory will therefore relate to the point-trading model employed in the empirical analysis.


Fig. 1. Equilibrium market prices and shipment pattern with trade between two regions.

## Minimizing Total Costs

The problem to be faced is: Given a level of supply of raw product and a level of demand for final product in each region, what conditions are necessary and sufficient for minimizing the total costs of processing and transportation under the equilibrium restraint that total supply equals total demand?

Consider the general case. Each region, $i$, has a given supply of raw product, $\boldsymbol{Q}_{i}$. The processing cost in each region $\left(P C_{i}\right)$ is a function of the level of processing, $Q_{i}^{*}$, and may be represented by $P C_{i}=f_{i}\left(\boldsymbol{Q}_{\boldsymbol{i}}^{*}\right)$. Conceptually, $n$
$Q_{i}^{*}$ can vary from 0 to $\Sigma \boldsymbol{Q}_{i}$. Raw product

$$
i=1
$$

assembly cost ( $A C_{i}$ ) for shipments from other regions is a function of the difference between quantity of raw product in region $i$ and the level of processing in that region, or

$$
A C_{i}=g_{i}\left(Q_{i}-Q_{i}\right)
$$

Final product distribution costs from each region to other regions $\left(D C_{i}\right)$ can be described in terms of the difference between the level of processing and the demand for the final product in region

$$
\stackrel{i,\left(D_{i}\right), \text { or }}{D C_{i}=h_{i}\left(Q_{i}^{*}-D_{i}\right) .}
$$

In the simplest case, $A C_{i}=$ transportation cost per unit from region $j$ to region $i$ times the total shipment from $j$ to $i$. For example, for two regions, $A C_{1}=$ ( $c_{21}$ ) ( $\left.\boldsymbol{Q}_{1}^{*}-\boldsymbol{Q}_{1}\right)$ with $c_{21}$ equal to cost per unit moved and $Q_{1}^{*}-Q_{1}$ equal to total units moved. Here the $c_{21}$ is a constant; in the more complicated case, $c_{21}$ could be a function of $\boldsymbol{Q}_{1}^{*}-\boldsymbol{Q}_{1}$. The same situation holds for the distribution costs.

If we consider these functions for a simple two-region case, then the total costs of processing and transportation, given $Q_{1}$ and $Q_{2}$, would be

$$
\begin{aligned}
T C= & P C_{1}+P C_{2}+A C_{1}+A C_{2} \\
& +D C_{1}+D C_{2}+\lambda\left[\left(Q_{1}+Q_{2}\right)\right. \\
& \left.-\left(Q_{1}^{*}+Q_{2}^{*}\right)\right],
\end{aligned}
$$

or

$$
\begin{aligned}
T C= & f_{1}\left(Q_{1}^{*}\right)+f_{2}\left(Q_{2}^{*}\right)+g_{21} \\
& \left(Q_{1}^{*}-Q_{1}\right)+g_{12}\left(Q_{2}^{*}-Q_{2}\right) \\
& +h_{12}\left(Q_{1}^{*}-D_{1}\right)+h_{21} \\
& \left(Q_{2}^{*}-D_{2}\right)+\lambda\left[\left(Q_{1}+Q_{2}\right)\right. \\
& \left.-\left(Q_{1}^{*}+Q_{2}^{*}\right)\right]
\end{aligned}
$$

where $\lambda\left[\left(\boldsymbol{Q}_{1}+\boldsymbol{Q}_{2}\right)-\left(\boldsymbol{Q}_{1}^{*}+\boldsymbol{Q}_{2}^{*}\right)\right]$ is the restraint that total processing output must equal total supply of raw product (in equivalent units), or that final demand equals initial supply.

Assuming continuous functions with continuous first and second derivatives, the first order conditions for minimizing total cost with respect to processing levels in the two regions are the following:

$$
\begin{aligned}
\frac{\partial T C}{\partial Q_{1}^{*}}= & f_{1}^{\prime}\left(Q_{1}^{*}\right)+g_{21}^{\prime}\left(Q_{1}, Q_{1}^{*}\right) \\
& +h_{12}^{\prime}\left(Q_{1}^{*}, D_{1}\right)-\lambda=0 \\
\frac{\partial T C}{\partial Q_{2}^{*}}= & f_{2}^{\prime}\left(Q_{2}^{*}\right)+g_{12}^{\prime}\left(Q_{2}, Q_{2}^{*}\right) \\
& +h_{21}^{\prime}\left(Q_{2}^{*}, D_{2}\right)-\lambda=0 \\
\frac{\partial T C}{\partial \lambda}= & \left(Q_{1}+Q_{2}\right)-\left(Q_{1}^{*}+Q_{2}^{*}\right)=0
\end{aligned}
$$

or

$$
f_{1}^{\prime}\left(Q_{1}^{*}\right)+g_{21}^{\prime}\left(Q_{1}, Q_{1}^{*}\right)+h_{12}^{\prime}\left(Q_{1}^{*}, D_{1}\right)
$$

$$
\begin{aligned}
& =f_{2}^{\prime}\left(Q_{2}^{*}\right)+g_{12}^{\prime}\left(Q_{2}, Q_{2}^{*}\right) \\
& +h_{21}^{\prime}\left(Q_{2}^{*}, D_{2}\right)=\lambda .
\end{aligned}
$$

Thus, to minimize total costs with respect to output in each region, it is necessary that the sums of the marginal costs of processing, assembly, and distribution in each region be equal. The second order conditions for a minimum stipulate that the second derivatives be positive, or
$f_{1}^{\prime \prime}\left(Q_{1}^{*}\right)+g_{21}^{\prime \prime}\left(Q_{1}, Q_{1}^{*}\right)+h_{12}^{\prime \prime}\left(Q_{1}^{*}, D_{1}\right)>0$
$f_{2}^{\prime \prime}\left(Q_{2}^{*}\right)+g_{12}^{\prime \prime}\left(Q_{2}, Q_{2}^{*}\right)+h_{21}^{\prime \prime}\left(Q_{2}^{*}, D_{2}\right)>0$.
In other words, the sums of the slopes of the marginal cost functions within a region must be greater than zero, or the marginal curve to the regional total cost curve must be upward sloping. ${ }^{3}$

The above development can be extended to $n$ regions easily. In this case the total cost function is
$T C=\sum_{i=1}^{n} f_{i}\left(Q_{i}^{*}\right)+\sum_{j=1}^{n} \sum_{i=1}^{n} g_{i j}\left(Q_{i}, Q_{i}^{*}\right)+\sum_{i=1}^{n} \sum_{j=1}^{n} h_{i j}\left(Q_{i}^{*}, D_{i}\right)+\lambda\left(\sum_{i=1}^{n} Q_{i}-\sum_{i=1}^{n} Q_{i}^{*}\right)$.
The first order conditions are:

$$
\begin{array}{cccc}
\frac{\partial T C}{\partial Q_{1}^{*}}= & V_{1}=f_{1}^{\prime}\left(Q_{1}^{*}\right)+\sum_{j=1}^{n} g_{j 1}^{\prime}\left(Q_{1}^{*}, Q_{1}\right)+\sum_{j=1}^{n} h_{1 j}^{\prime}\left(Q_{1}^{*}, D_{1}\right)-\lambda= & 0 \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\frac{\partial T C}{\partial Q_{n}^{*}}= & V_{n}=f_{n}^{\prime}\left(Q_{n}^{*}\right)+\sum_{j=1}^{n} g_{j n}^{\prime}\left(Q_{n}^{*}, Q_{n}\right)+\sum_{j=1}^{n} h_{n j}^{\prime}\left(Q_{n}^{*}, D_{n}\right)-\lambda=0
\end{array}
$$

[^1]The second order conditions are that the bordered principal minors of the Hessian determinant of second partial
$\left|\begin{array}{ccc}V_{11} & V_{12} & -1 \\ V_{21} & V_{22} & -1 \\ -1 & -1 & 0\end{array}\right|<0 \cdots\left|\begin{array}{cccc}V_{11} & V_{12} \cdots & V_{1 n} & -1 \\ V_{21} & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot \\ V_{n 1} & \cdots & V_{n n} & -1 \\ -1 & \cdots & -1 & 0\end{array}\right|<0$.

If the functions involved are not continuous functions with continuous first and second derivatives, the above approach must be reformulated. Frequently the cost functions or transportation rates for a given distance will be linear in nature, an intercept term may be involved, or the functions will be relevant only for certain ranges of quantities (for instance, assembly costs may not be relevant until the local supply of raw product is exhausted). In these cases, the summation of the various functions results in a kinked or, possibly, a step function rather than the smooth, continuous curves to which calculus can be applied. Indeed, minimization via the above procedures will not yield a unique point as will be discussed later.

Before considering the reformulation of conventional theory for minimizing costs of processing and transportation (and thus determining the size and location of plants), the nature of the demand, supply, and transfer cost functions applicable to the immediate analysis may be noted briefly.
derivatives all be negative for a minimum, or ${ }^{4}$
 tion for a single homogeneous product requires no special comment, except to note its conventional nature-namely, that the quantity consumed is an inverse function of price, holding prices of competing goods, income, and other factors constant. The supply function (fig. 1) combines the raw product supply function with the processing supply function. We specify raw product assembly costs to be zero within the region, and similarly for final product delivery and associated distribution costs of wholesalers and retailers. Interregional transfer costs will be developed in more detail subsequently. The product supply function for raw product is considered to be an increasing function of price in each region. Theoretical constructs relating to this proposition are readily available, and attention is directed to the processing supply function.

The processing supply function conventionally is specified as an increasing function of price, which follows directly from assumed U-shaped average cost functions for the firm and an increas-

[^2]ing-cost industry situation. Empirical estimation of processing firm economies of scale cost curves would indicate decreasing average costs over the entire relevant range of output for the singleplant firm, if plants are operated an 8hour shift with no overtime payments. With plants operated at capacity (minimum average cost), replication of plants of this size would assure supply functions for the long-run situation that are relatively more elastic than implied with the usual U-shaped cost function, even though the nature of the long-run curve for larger single plants may be upward sloping. In fact, a perfectly elastic supply function over large ranges of output for processing services does not appear unreasonable for an industry of minor importance in the regional economy. The total supply function, however, would have a positive slope due to the response in the production of raw product. ${ }^{5}$

Figure 2, section A, presents a hypothetical economies of scale curve for a processing operation. The level and/or slope of the curve may vary among regions, due to differences in such costs as construction, labor, and utilities. This relationship also is shown as a total cost function for each region for convenience in later arguments. Section A of figure 2 shows functions for a situation in which the scale of plant can increase smoothly and continuously. When considering expansion of processing facilities the use of multiple plants may offer the economically feasible means. Section B, figure 2 , shows relationships for longrun cost functions for multiple plants. The long-run total cost function will appear as a step function with an intercept value added with each additional
plant. When one plant is fully utilized, expansion is achieved by addition of another plant. ${ }^{6}$

The interregional final product shipment costs shown in figure 3 are specified to be a constant average cost per quantity for a given distance of shipment. This is a simple supply function for transportation services in the case where product shipments are a minor portion of total shipments by the transportation firm. Similar transfer cost functions could be shown for shipment of raw product. For this theoretical development, we assume that in terms of final product equivalent the cost for raw material equals that for final product.

Turning now to the location of processing facilities, we may illustrate this problem in a simple context in which regional supplies of raw product and product demand are specified at a given level in each region. Product demand (figure 4, section A ) equals $\mathrm{OD}^{\prime}$ in region 1 and $\mathrm{OD}^{\prime \prime}$ in region 2, giving a total of OT in the two regions. Product supply is $\mathrm{OS}^{\prime}$ in region 1 , and $O S^{\prime \prime}$ in region 2 with $\mathrm{OS}^{\prime}+\mathrm{OS}^{\prime \prime}=\mathrm{OD}^{\prime}+\mathrm{OD}^{\prime \prime}=$ OT. The question is: Should processing be done in one region or in two regions, and-if in two regions-how much should be processed in each region? Factors to consider are the following: (1) regional differences in the processing cost function; (2) the shape of the processing cost function; (3) the level of regional demand in relation to raw product supply; and (4) transfer cost function characteristics for final product and for raw product.

Figure 4 illustrates a particular situation for these variables. Section A presents total costs of processing and shipment of final product and/or raw prod-

[^3]

Fig. 2. Average and total long-run cost functions for single plants (section A) and for multiple plants (section B).
uct under alternative levels of processing in each region. To illustrate for region 1, if this region processes more than domestic demand ( $\mathrm{OD}^{\prime}$ ), this will require product shipments to region 2. Thus, the product shipment transfer function is added to the processing cost function at this level of processing output. Similarly, if region 1 processes more than the domestic supply of raw
product ( $\mathrm{OS}^{\prime}$ ), then raw product must be shipped in from region 2. Thus, the raw product shipment cost is added to the function at output level $\mathrm{OS}^{\prime}$. A similar function is developed for region 2.

Determination of total costs of processing, if all processing is done in one region, requires only the comparison of total costs at output level OT. In


Fig. 3. Interregional final product shipment costs, average and total.
this case, costs would be lower at region 1 . The more interesting case involves the determination of levels of cost and output of processing done in both regions. The pattern of processing and shipments which minimizes total costs in this case cannot be derived through use of calculus.

The first order conditions are that the sums of the slopes of the individual cost functions for each region must be equal. However, the second order conditions that the second derivatives be positive is not met since the second derivatives of the linear functions are equal to zero. Thus, we cannot say that equating the slopes will be a minimum. Also, since the first order conditions indicate equal slopes of the area total cost functions are necessary for a minimum, and since the slopes of linear functions are constant, it may be that there is no point at which the area slopes are equal, or there may be no unique point-that is, they may be equal for all quantities.

Since it may be less costly to produce in one plant, examination of total rather than marginal costs is required. In the illustrated case where the total cost curves for each region are straight lines, the marginal cost curves are discontinuous (fig. 4, section B.) Section C of figure 4 shows the total costs asso-
ciated with varying levels of processing the quantity OT in region 1 .

If no processing is done in region 1 , total costs equal OW , with all processing done in region 2. If all processing is done in region 1, total costs equal TZ. If processing is done in both regions, the long-run fixed costs for two plants are required, and thus the upward shift in the cost function shown in section C for all outputs other than those mentioned above. The minimum cost situation of $S^{\prime} Y$ is associated with each region processing that volume equal to its raw product supply ( $\mathrm{OS}^{\prime}$ and $\mathrm{OS}^{\prime \prime}$ ).

Other specific cost situations may also require evaluation of total cost alternatives rather than the use of marginal concepts used in conventional theory. For example, if the processing cost function included a large enough intercept value, it can be shown that costs would be lowest by processing at only one location. If the raw product supply in the region is small, average processing costs are apt to be excessively high in relation to other regions with large supply sources. Similarly, if the processing costs vary between regions to an important extent (either because of different intercept values or slopes), this can influence location. In addition, the distance of source to processing point


Fig. 4. Cost-minimizing plant location for two regions with given levels of raw product supply, product demand, processing costs, and transfer costs.
will influence the transfer cost function (fig. 3) and will affect location.

For a particular set of conditions, it is possible to specify the least-cost location for processing facilities. Results developed for the two-region case may be generalized to the multiregion case.

## MODEL OF OPTIMUM SIZE AND LOCATION

The framework for this analysis is based upon location and trade theory relating to a single homogeneous product. The theoretical constructs used in this study draw upon previous theoretical and empirical analyses, as will be noted in the following sections. The empirical analysis is concerned with the following problem: Various regions in California have given supplies of live animals and/or given demand for meat. The transportation costs for both live animals and dressed beef are given and do not vary with quantity shipped. ${ }^{7}$ Slaughtering costs for animals vary by region, and economies of scale exist in slaughtering facilities. The questions to be answered are: Where should processing plants be located? What should be the capacity of each plant? What is the optimum number of plants needed to move the animals through slaughter plants to consumers at least aggregate cost?

The model thus considers a system of plants and shipment patterns that minimize the cost of assembling, processing, and distributing the final product. The regional supply functions for the raw material (live animals) are considered as completely inelastic at given levels. The regional demand functions for the product (beef) are also considered inelastic at given levels. The framework for appraising location and size of plants under this model specification will be developed next, followed by the linear
programming model used in the analysis.

When the alternative supply sources and demand areas are few in number or are treated as such, the determination of location and size of plant may be solved by budgeting methods as shown by Isard, Schooler, and Vietorisz (1959). Less aggregative analyses, involving spatially separated markets for raw product and final product, make budgeting procedures complex, especially when consideration is given to economies of scale in processing plant costs.

With the specification that the regional supply functions of the live animals and the demand functions for the carcass beef are completely inelastic (unresponsive to price changes), a modified form of the transhipment model of linear programming provides a convenient computational tool for deriving optimum plant location. An iterative procedure is required to obtain a final and approximate solution when economies of scale (or diseconomies) are present in long-run average costs.

Because of area aggregation and the use of central points representing these aggregated regions, the model is a pointtrading variety rather than a continuous market supply-area model, and is similar to the transportation or spatial equilibrium models. This factor also imposes spatial monopoly aspects necessarily present within the regions (Bressler, 1958).

## TWO ALTERNATIVE MODELS

The logic of the location problem is seen most readily if it is stated as a product-factor spatial equilibrium model. This model will be developed first, followed by a restatement of the problem as a modified transhipment model noting the computational advantage of such a formulation.

[^4]
## Raw Product-Final Product Spatial Equilibrium Model ${ }^{8}$

For simplicity, assume initially that there are no economies of scale in slaughter operations but that costs vary by region. Supplies of slaughter animals and the quantity consumed by region are given. The problem is to minimize the combined cost of assembly of live animals, slaughter, and shipment of meat to consuming regions. The solution provides the optimum volume of slaughter at each location. With no economies of scale in processing, we may determine the number of plants of given capacity required to process this volume. We will relax the assumption as to economies of scale presently. This problem may be expressed in mathematical terms as follows:

Minimize:

$$
\begin{align*}
& \sum_{i j} \sum_{i j} T_{i j} X_{i j}+\sum_{i} H_{i} S^{i} \\
& +\sum_{i j} \sum_{i j} L_{i j} \tag{1.1}
\end{align*}
$$

(total cost of meat shipment, slaughter, and animal shipment)

Subject to:

$$
\begin{equation*}
\sum_{j} X_{i j}=\alpha_{i} S^{i} \tag{1.2}
\end{equation*}
$$

(meat shipments equal meat equivalent of animals slaughtered)

$$
\begin{equation*}
S^{i}=S_{i}-\sum_{j}\left(L_{i j}-L_{j i}\right) \tag{1.3}
\end{equation*}
$$

(number slaughtered equals supply adjusted for shipments)

$$
\begin{equation*}
\sum_{j} X_{j i}=D_{i} \tag{1.4}
\end{equation*}
$$

(shipments to a region equal quantity consumed)

$$
\begin{gather*}
0 \leq X_{i j}, L_{i j}, S^{i}  \tag{1.5}\\
\sum_{i} D_{i}=\sum_{i} \alpha_{i} S^{i}=\sum_{i j} \sum_{i j} X_{i j} \tag{1.6}
\end{gather*}
$$

(quantity consumed equals total slaughter equals total meat shipment)
where

$$
\begin{aligned}
& X_{i j}=\begin{array}{c}
\text { meat shipment from region } i \text { to } \\
\\
\text { region } j
\end{array} \\
& L_{i j}=\text { live animal shipment from } \\
& \text { region } i \text { to region } j
\end{aligned}
$$

Table 1 gives the linear programming format for an illustrative two-region case. Such a factor-product model for, say, 30 regions would require some 90 equations and 1,800 activities ( 900 for meat shipment, 30 for slaughter, and 870 for live animal shipment since shipments within a region are not required). We must also consider adjustments

[^5]Table 1

## A TWO-REGION EXAMPLE OF THE SLAUGHTER PLANT LOCATION USING A FACTOR-PRODUCT SPATIAL EQUILIBRIUM MODEL*

| Equation | Activity |  |  |  |  |  |  |  | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Meat shipment |  |  |  | Livestock slaughter |  | Live animal shipment |  |  |
|  | $X_{11}$ | $X_{12}$ | $X_{21}$ | $X_{22}$ | $S^{1}$ | $S^{2}$ | $L_{12}$ | $L_{21}$ |  |
| 0. | $T_{11}$ | $T_{12}$ | $T_{21}$ | $T_{22}$ | $H_{1}$ | $\mathrm{H}_{2}$ | $t_{12}$ | $t_{21}$ | $\ldots$ |
| 1. | 1 | 1 | $\ldots$ | $\ldots$ | -a | $\ldots$ | $\ldots$ | ... | $=0$ |
| 2. | ... | ... | 1 | 1 | ... | -a | $\ldots$ | $\ldots$ | $=0$ |
| 3. | $\ldots$ | ... | $\ldots$ | ... | 1 | ... | 1 | -1 | $=S_{1}$ |
| 4. | $\cdots$ | ... | $\ldots$ | ... | $\ldots$ | 1 | -1 | 1 | $=S_{2}$ |
| 5. | 1 | $\cdots$ | 1 | $\cdots$ | $\ldots$ | ... | ... | $\ldots$ | $=D_{1}$ |
| 6. |  | 1 | $\ldots$ | 1 | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $=D_{2}$ |

* Notation used is as follows ( $i$ and $j=1$ and 2):
$X_{i j}=$ meat shipment from region $i$ to region $j$
$X_{i j}=$ meat shipment from region $i$ to region $j$
$L_{i j}=$ live animal shipment from region $i$ to region $j$
$\boldsymbol{S}_{\boldsymbol{T}}^{\boldsymbol{i}}=$ slaughter of cattle in region $\boldsymbol{i}$
$T_{i j}=$ meat transfer cost from region $i$ to region $j$
$t_{i j}=$ animal transfer cost from region $i$ to region $j$
$H_{i}=$ slaughter cost per head in region $j$
$a_{i}=$ dressing percentage in region $i$
$S_{i}=$ supply of slaughter cattle in region $i$
$D_{i}=$ demand for meat in region $i$
allowing for economies of scale in processing costs.


## Consideration of Economies of Scale

Previous studies have included consideration of economies of scale in the location of processing plants, but these have been confined to assembly costs and processing costs. ${ }^{10}$ In our problem, we must also consider product shipment costs. The general approach used in this study was to assign to each region a value of processing costs $\left(H_{i}\right)$ associated with a large processing plant; this cost is the lowest possible for that region. The problem is run to determine the volume of processing under these conditions in each region. Second, the volume of processing at each location is then compared with the actual cost of processing that volume, based on an economies of scale curve for specialized beef slaughter plants. Third, processing costs are revised and the problem rerun.

Costs for regions in which no processing took place under the most favorable cost situation are raised to that level associated with a small volume of slaughter, effectively eliminating them from consideration in the second approximation. Costs in other regions are adjusted to be in line with slaughter volume. Fourth, results from the second solution are inspected for possible adjustments and further adjustments made. Thus, an iterative approach is used to solve, by linear techniques, a nonlinear programming problem. ${ }^{11}$

Revising slaughtering costs to make them compatible with the volume of animals in a particular region may, in some cases, be misleading. For example, total assembly, processing, and distribution costs may be lowered for a group of small supply regions by combining their supplies of live animals at one central point, thus utilizing a larger plant than would be indicated by the volume of animals in any one of the regions. How-

[^6]ever, if the costs of processing in the programming problem are those levels associated with the volume found in each region rather than the combined volume, then the establishment of a plant in that area of the state will likely be neglected. In these cases, a budgeting procedure may be used to estimate the feasibility of lowering the costs in a central region to allow for the possibility of a larger plant drawing on the combined supplies of surrounding regions.

## Modified Transhipment Model

Orden (1956) has shown that the basic transportation model may be modified to allow shipments of a good to go by any sequence of points rather than just from $m$ surplus regions to $n$ deficit regions. Basically, the transportation model is modified by specifying each production and consumption area as a possible shipment or transhipment point. The possible use of this model became most apparent in an article by Kriebel (1961) dealing with warehousing with transhipment under seasonal demand. His problem deals with the shipment of a good from a producing center (or centers) to a consuming center (or centers) directly in a given time period or by shipment to a warehouse for transhipment immediately, or for storage for one or more periods. This is similar in some respects to our problem if we consider storage costs to be equivalent to slaughter costs. We ask: Should animals be slaughtered at the source and meat shipped, or should animals be shipped and slaughtered at one of several possible points and then the meat shipped to the demand areas? The slaughter costs will depend on the number of animals shipped to that slaughter point.

The nature of the solution is most readily apparent with a simple three-
region example given in figure 5 . In this case, each region has a supply of live animals and each region has a demand for the final product. No distinction is made as to whether the region is surplus or deficit. Therefore, whereas the usual transportation models allow for shipments from $m$ surplus regions to $n$ independent deficit regions, the example includes $m+n$ shipping regions and $m+n$ receiving areas.

Section A gives the matrix format which consists of four submatrices each of dimension $N$ (where $N=m+n$ ). ${ }^{12}$ Live animal supplies ( $k_{i}$ ) are expressed in terms of meat equivalent. Costs are given in terms of units of meat equivalent also. Submatrix $L$ gives the cost of live animal shipments. Thus, the shipment from region 1 to region 2 is $\$ 2.00$ per unit. In this problem, an activity would be included in the final solution only if it were cheaper to ship from one source by transhipment through another source to a final destination, rather than directly from the origin to final destination.

Submatrix $L+S$ in section B gives the cost of live animal shipment, plus the cost of slaughter in region $j$; that is, slaughter cost is added to shipment cost to obtain the listed entries. In the example, the cost of slaughter is $\$ 5.00$ in region $1, \$ 7.00$ in region 2 , and $\$ 8.00$ in region 3. These costs appear on the main diagonal, exhibiting zero shipment costs within a region.

Submatrix $Y$ has no relevance to the problem and would enter only if cost of shipping meat from $i$ to $j$ differed from the cost of shipping meat from $j$ to $i$. To avoid entries, high transfer costs are introduced in the cost matrix, as shown in section $B$.

Submatrix $M$ provides for meat shipment from slaughter points to the consuming points. Both for live animal shipment and for meat, the within-

[^7]
## Section A: Matrix format



Section C: Minimum cost solution


Section B: Cost matrix


Section D: Final solution


Fig. 5. A three-region example of the slaughter plant location model, using a modified transhipment formulation.
region transportation costs per 100 pounds are assumed to be equal for all regions, and thus set equal to zero.

The supplies of live animals in meat equivalent ( $k_{i}$ ) for supply areas $1-3$ are 15,8 , and 5 , respectively. The meat re-
quirements $\left(r_{j}\right)$ for areas 1-3 are 7, 14, and 7, respectively. A key to the solution of this problem is the introduction of an artificial variable $A$ which is taken to be equal or greater than the sum of the requirements $r_{j}$. In this problem, a
value of 30 was used for illustration. As noted in figure 5 , section $A$, this value is added to each row and column of the $2 N$ matrix. The introduction of this variable allows transhipment of live animals through a slaughter point and on to a demand center, as will be illustrated for region 1 in the example. Use of a magnitude equal to total consumption in all regions allows processing the total quantity at one slaughter location.

The minimum cost solution of our sample problem is given in section $C$. The main diagonal consists of entries of the $A$ value of 30 except where transhipment occurs. Entries in the $L+S$ matrix indicate levels of slaughter of 15 in region 1,8 in region 2 , and 5 in region 3 . Entries in the $M$ matrix indicate meat shipments from other slaughter regions to deficit regions. For example, region 1 ships 6 units to region 2 and 2 units to region 3, for a total of 8 units. Due to these transhipments the value in the main diagonal for region 1 differs from 30 by these eight units. The final solution to the problem (given in section D ) subtracts the value of $A$ of 30 for each row and column. In actual computations, the next step would be to correct the slaughter costs for volume slaughtered by the iterative technique suggested previously.

The computational advantage of the transhipment formulation is clear. For a 30 -region case, a matrix of $2 N$ is required, or 60 by 60 . This compares with the alternative formulation requiring 90 equations and 1,800 activities. However,
the latter method of solution is more flexible if it is desirable to relax the assumption of a completely inelastic demand function. The model to be described for location of California slaughter plants involved over 30 regions and was solved on an IBM 1620 (capacity = 60,000 digits) using the conventional transportation problem program.

The mathematical formulation of the model where each supply and/or demand region is a possible slaughter region may be stated as follows:

Minimize:

$$
\begin{equation*}
Z=\sum_{i=1}^{2 N} \sum_{j=1}^{2 N} c_{i j} x_{i j} \tag{2.1}
\end{equation*}
$$

(total cost of all shipments of meat, and shipment and slaughter of live animals in terms of meat equivalent)

Subject to:

$$
\begin{align*}
\sum_{i} x_{i j} & =r_{j}+A \text { for } i, j  \tag{2.2}\\
& =1 \cdots N, N+1 \cdots 2 N \\
N & =m+n
\end{align*}
$$

(total meat requirements at consuming area must equal total shipments to area, plus artificial value $A$ )

$$
\begin{equation*}
-\sum_{j} x_{i j}=-k_{i}-A \tag{2.3}
\end{equation*}
$$

(total shipments from an area must equal total supply, plus artificial value $A$ )
$x_{i j} \geq 0$ where $x_{i j}$ are the meat equivalent of live animals for $i=1 \cdots N$;
$j=1 \cdots N, N+1 \cdots 2 N$; and where $x_{i j}$ are meat shipments for $i=N+1 \cdots 2 N ; j=1 \cdots N, N+1 \cdots 2 N$.
$c_{i j}>0$ for $i \neq j$
$=0$ for $i=j$ and $i, j=1 \cdots N, N+1 \cdots 2 N$. (within region transporation cost is zero)
$=H_{i}$ for $i=j$ and $i=1 \cdots N ; j=N+1 \cdots 2 N$. (diagonal of one submatrix contains slaughter costs)
$=\infty$ for $i=j$ and $i=N+1 \cdots 2 N ; j=1 \cdots N$.
(diagonal and entire cost matrix contains high cost to preclude entries in shipment pattern)

$$
\begin{align*}
& A \geq \sum_{j=N+1}^{2 N} r_{j} \text { where } r_{j}=0 \text { for } j=1 \cdots N .  \tag{2.6}\\
& \quad \begin{array}{l}
\text { (artificial value } A \text { is chosen to assure inclusion of only relevant } \\
x_{i j} \text { in the optimum solution) }
\end{array} \\
& k_{i}=0 \text { for } i=N+1 \cdots 2 N . \\
& \quad \begin{array}{c}
\text { (supply of live animals in meat equivalent is limited to relevant } \\
\text { matrix) }
\end{array} \tag{2.7}
\end{align*}
$$

## PRICE DIFFERENTIALS

Once the optimum pattern of shipments is determined a set of relative prices can be calculated for the shipping and receiving areas. Such calculation draws on the theoretical concept that prices between shipping and receiving areas will differ by exactly the cost of transportation in competitive equilibrium. Because of the dual nature of the transportation model, the prices which were derived from minimization of the
transfer costs are the same as those which would exist if the firms were trying to sell their output at the maximum price under pure competition. Thus, had the problem been set up to maximize prices received by the slaughtering firms (and the producers of live animals) with the constraint that the prices could not exceed the value of the product at the plant plus transportation, the shipment pattern and the set of relative prices would be the same. ${ }^{13}$

## SUPPLY, DEMAND, TRANSFER, AND PROCESSING COST FUNCTIONS

As outlined previously, the optimum size and location of plants depend basically on four criteria-the supply of raw materials, the demand for the final product, the transportation or transfer costs, and the nature of the processing cost function. These four functions then serve as the basic data to be used in deriving the location and scale of plants.

## RAW PRODUCT SUPPLY

Animals for slaughter in California originate from several sources: feedlots, dairy herds, beef herds, grass-fed cattle, and inshipments of animals for immediate slaughter from neighboring states. While the number of cattle supplied by these sources is not available by county, information on a state-wide level is published from which county supplies can be estimated. Table 2 summarizes total annual slaughter, inshipments of stockers and feeders, inship-
ments of cattle and calves for immediate slaughter, and outshipments of cattle and calves, for 1960.

Table 2
CATTLE AVAILABLE FOR SLAUGHTER FROM VARIOUS SOURCES, 1960


[^8][^9]The total number of dairy cows on California farms in 1960 was estimated by adding the number of cows 2 years old or older on farms January 1, 1960899,000 head-and the number of heifers 1 and 2 years old on farms January 1, 1960-250,000 head. ${ }^{14}$ These heifers, it was assumed, would reach the category of "milk cows" sometime during 1960. A death loss of 3.7 per cent was then applied to these figures. From the total number of cows on farms was subtracted the death loss and the number of cows on the farms at the beginning of 1961 -also 899,000 head. The resulting 208,000 head then became the number available for slaughter.

A similar procedure was followed to estimate the number of beef cows available for slaughter. At the beginning of 1960, there were 853,000 beef cows 2 years old or older and 358,000 heifers. Using the same death rate, the number available for slaughter was estimated at 315,000 head.

On January 1, 1960, there were 846,000 steers and bulls (beef type) 1 year old or older on California farms, and there were 630,000 calves. Half of these animals were assumed to be bull calves and to reach the year-old level in 1960 , and the other half was assumed to be heifer calves. Again the 3.7 per cent death rate was used, resulting in a level of 179,000 head of beef steers and bulls available for slaughter. Of the calf crop the number raised, $1,449,000$, minus the number at the end of the year, 960,000 , left 489,000 head available for slaughter.

The total disappearance of cattle and calves during 1960 equalled $2,940,000$ head (see table 2), but the total slaughter of cattle and calves for the year was $2,958,000$ head. It was assumed that the additional 18,000 head were primarily dairy bulls and steers, a category not included in any of the above. (Some of the animals included in this group may
also represent errors in the estimation of the other categories.)

There were 482,000 calves slaughtered in 1960. Since the disappearance was 489,000 calves, it was assumed that 7,000 head of calves were included in the outshipments. The state is self-sufficient with respect to demand for calves for slaughter; inshipments of animals for immediate slaughter were assumed to be cattle. Since the California calf crop exceeds the number slaughtered, this analysis will be concerned only with the slaughter and shipment of cattle.

Cattle available for slaughtering were divided into three classes: (1) marketings from feedlots, (2) dairy animals, and (3) cull beef and grass-fed cattle. There were 226,000 dairy animals available for slaughter; beef cattle total $1,800,000$ head after subtracting outshipments. Of this last number, $1,595,-$ 000 head were marketed from feedlots, leaving 205,000 head as cull beef and grass-fed animals.

Dairy stock was allocated to the counties by computing the percentage of milk cows, 2 years old and older, found in each county in 1960 and then applying that figure to the total of 226,000 head. Data on marketings from feedlots by county for 1961 were obtained on a confidential basis from the California Crop and Livestock Reporting Service. Derivations of county marketings as a percentage of the state total were made and applied to the total marketings in 1960. The remaining classification, cull beef and grass-fed cattle, was separated into county supplies by use of data from U. S. Bureau of the Census (1961a). The number of milk cows by county for 1960 was subtracted from the total number of cows (beef and dairy) to estimate the quantity of beef cows by county. The percentage of the state total of beef cows in each county was assumed to be the same as the percentage of cull beef and grass-fed cattle available for slaughter in each county. Table 3 gives

[^10]the total supply of cattle for slaughter by county.

The supply of cattle for slaughter was converted to a dressed weight basis, using average 1960 weights of 1,026 pounds per head and a dressing percentage of 57.3 per cent given by the U. S. Department of Agriculture (1961). Average dressed weight was 587.9 pounds. In addition to the dressed carcass, however, the edible by-products-liver, heart, lungs, tongue, and cheek meat-must also be considered as part of the supply of beef available. These by-products equal approximately 3.7 per cent or 38 pounds
of the live weight of the animal, according to Vaughn (1961), and increase the total weight of the edible meat from the average animal to 626 pounds.

Table 3 gives the production of live animals in terms of pounds of dressed beef ( 626 pounds times the supply of cattle).

## Inshipments

Of the 450,000 head shipped into the state for immediate slaughter in 1960, Arizona supplied 253,000; Colorado, 12,000; Idaho, 57,000; Nevada, 21,000; Oregon, 17,000; Texas, 32,000; Utah, 36,000 ; and other areas, 22,000 head

Table 3
ESTIMATED PRODUCTION AND DERIVED CONSUMPTION OF BEEF BY COUNTY, 1960

| County | Production of live animals* | Production of dressed beef $\dagger$ | Consumption of dressed beef $\ddagger$ | County | Production of live animals* | Production of dressed beef $\dagger$ | Consump tion of dressed beef $\ddagger$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | number | 1,000 pounds |  |  | number | 1,000 pounds |  |
| Alameda | 9,263 | 5,799 | 109,486 | Sacramento. | 57,265 | 35,848 | 60,608 |
| Alpine | 226 | 141 | 48 | San Benito. | 10,727 | 6,715 | 1,856 |
| Amador | 1,900 | 1,189 | 1,204 | San Bernardino. | 16,558 | 10,365 | 60,706 |
| Butte. | 8,517 | 5,331 | 9,888 | San Diego. | 9,888 | 6,190 | 124,530 |
| Calaveras. | 3,307 | 2,070 | 1,240 | San Francisco. | ...... | 0 | 89,245 |
| Colusa. | 2,818 | 1,764 | 1,456 | San Joaquin | 53,996 | 33,801 | 30,135 |
| Contra Costa | 17,502 | 10,956 | 49,307 | San Luis Obispo | 23,380 | 14,636 | 9,770 |
| Del Norte | 1,255 | 786 | 2,142 | San Mateo. | 1,125 | 704 | 53,570 |
| El Dorado. | 1,607 | 1,006 | 3,543 | Santa Barbara. | 33,003 | 20,660 | 20,368 |
| Fresno. | 178,819 | 111,941 | 44,114 | Santa Clara. | 19,942 | 12,484 | 77,429 |
| Glenn. | 19,662 | 12,308 | 2,079 | Santa Cruz. | 2,270 | 1,421 | 10,152 |
| Humboldt. | 9,690 | 6,066 | 12,644 | Shasta | 6,773 | 4,240 | 7,169 |
| Imperial. | 384,790 | 240,879 | 8,692 | Sierra. | 947 | 593 | 271 |
| Inyo. | 3,254 | 2,037 | 1,408 | Siskiyou | 13,593 | 8,509 | 3,964 |
| Kern. | 231,199 | 144,731 | 35,198 | Solano. | 31,298 | 19,593 | 16,225 |
| Kings | 73,011 | 45,705 | 6,022 | Sonoma | 14,974 | 9,374 | 17,766 |
| Lake. | 1,452 | 909 | 1,662 | Stanislaus | 43,178 | 27,029 | 18,961 |
| Lassen | 8,978 | 5,620 | 1,639 | Sutter | 2,582 | 1,616. | 4,024 |
| Los Angeles. | 144,836 | 90,667 | 727,960 | Tehama | 14,362 | 8,991 | 3,050 |
| Madera. | 51,894 | 32,486 | 4,878 | Trinity | 1,193 | 747 | 1,170 |
| Marin | 8,131 | 5,090 | 17,699 | Tulare | 126,268 | 79,044 | 20,300 |
| Mariposa. | 1,993 | 1,248 | 610 | Tuolumne. | 1,735 | 1,086 | 1,736 |
| Mendocino. | 4,268 | 2,672 | 6,155 | Ventura | 51,404 | 32,179 | 24,005 |
| Merced | 63,542 | 39,777 | 10,903 | Yolo. | 18,474 | 11,565 | 7,923 |
| Modoc | 12,790 | 8,007 | 1,002 | Yuba | 7,398 | 4,631 | 4,082 |
| Mono | 1,089 | 682 | 267 |  | - | - |  |
| Monterey . | 31,954 | 20,003 | 23,911 | Total state | 2,026,000 | 1,268,276 |  |
| Napa. | 18,160 | 11,368 | 7,943 | Inshipments |  |  |  |
| Nevada. | 1,807 | 1,131 | 2,521 | cattle | 450,000 | 281,700 |  |
| Orange. | 12,440 | 7,787 | 84,858 |  |  |  |  |
| Placer | 6,173 | 3,864 | 6,871 | beef. |  | 344,700 |  |
| Plumas | 1,626 | 1,018 | 1,401 |  |  |  |  |
| Riverside | 145,714 | 91,217 | 36,910 | TOTAL | 2,476,000 | 1,894,676 | 1,894,676 |

[^11](Calif. Crop and Lvstk. Rptg. Serv., 1961). Truck shipments of both live animals and meat entering the state must pass through one of 17 border inspection stations. Two of these stations were used to represent the northern and southern entry points-Truckee and Blythe, respectively-for both live animals and dressed beef. Two central sources for inshipments of live animals were selected to represent transportation distances from out of state. Twin Falls, Idaho, represented live animal sources for shipments entering northern California, while southern inshipments came from Phoenix, Arizona.

Similar information on inshipments of meat is not published regularly. For 1961, meat and product inshipments were estimated to compose 18 per cent of the total beef supply in California, while the state's own livestock plus inshipments of cattle for immediate slaughter equalled 76 per cent of the total supply of dressed beef (King, 1962). (Imports from foreign countries equalled the other 6 per cent.) The dressed carcass weight of California slaughter in 1960, using a dressing percentage of 57.3 per cent, was $1,455,600$,000 pounds. ${ }^{15}$ From this figure-equal to 76 per cent of the total supply-inshipments were estimated to be $344,700,000$ pounds. Foreign exports and imports of meat were not considered because of lack of data.

The analysis of 1961 inshipments indicated that for truck inshipments of all meat some 45 per cent was directed to northern California and the remaining 55 per cent to southern California. Of the 892 million pounds of meat shipped into the state in 1961, therefore, 401.4 million pounds went to the Northern part and 490.6 million pounds were directed to the southern section. Some 38.9 per cent of the meat shipped into the northern section was beef; the southern proportion was 34.9 per cent (King,
1962). From these figures, estimations of the percentages of beef entering the state by regional destination were 47.7 per cent for the northern section and 52.3 per cent for southern California. These percentages were applied to 1960 inshipment estimates, with resulting supplies of $164,422,000$ pounds for northern California and 180,278,000 pounds for southern California.

From a limited sample of data it seems that 66 per cent of the inshipments into the northern part of the state originate in Idaho, Utah, Nevada, and Montana, while 63 per cent of those entering southern California came from Colorado. In aggregating the inshipments' supply points, it was assumed that the northern supplies would come from Salt Lake City, Utah, while those for southern California would be shipped from Denver, Colorado.

The total poundage of beef slaughtered in 1960 (including edible byproducts) plus inshipments of beef equalled $1,894,676,000$ pounds.

## CONSUMPTION OF DRESSED BEEF

To derive the consumption of beef and edible by-products by county, the production of beef in 1960 was set equal to the total consumption and divided by the state population to obtain a per capita level. The per capita consumption level of 120.457 pounds was then multiplied by the county population to estimate the total county consumption levels. ${ }^{16}$

## Slaughtering Regions

County boundaries offer a fairly simple means whereby regions within the state can be designated; however, the area represented by any one county is frequently quite different from those represented by other counties. In an effort to obtain meaningful regional patterns, the counties were combined

[^12]

Fig. 6. Supply of live animals and demand for meat for 32 regions in California and 2 inshipment points (1961).
into thirty-two supply and demand regions. ${ }^{17}$ The various regions and their supply of animals (in terms of dressed beef equivalent) are shown in figure 6.

## TRANSFER COSTS

Estimating the actual costs incurred by transporting live animals or meat would have required a complete cost analysis in that area. Such an analysis
would be a separate project in itself and is beyond the scope of this study. The State of California, through the Public Utilities Commission, sets minimum tariffs for hauling cattle and beef by truck within the state. According to the Commission, these rates are based on the cost of operation plus a return on investment for the hauling firms. While certain economies of vertical integration

[^13]of transportation-both assembling and distributing-might occur for slaughtering firms having their own transportation system, the rates set by the state appeared to offer the most reasonable cost estimates.

Both live animal and meat rates used were for 1960 and are given in terms of weight and constructive miles travelled. The constructive miles allow for topographical changes between points and for this reason should give a more accurate estimation of transportation costs than the use of road mileage. Rates for both cattle and beef vary with the minimum amount of weight to be hauled. In all cases, it was assumed that the shipments would qualify for the lowest rate, having minimum weights of 38,000 pounds for live animals and 20,000 pounds for the beef.

The transportation problem involves shipment of both meat and live animals, but to ease computational problems the live animal shipment costs and quantities shipped were expressed in dressed weight equivalent. Thus, the rate for shipping 38,000 pounds of live animals was adjusted by the ratio of live weight per head to dressed weight per head ( 1,026 pounds/ 626 pounds $=1.64$ ). This cost was then applied to the dressed weight equivalents of the live animal shipments.

The rates were also applied to inshipments of dressed beef and live animals on the following basis. The distances from Salt Lake City to Truckee and from Denver to Blythe were 560 miles and 925 miles, respectively. The simplification of having only two entry points instead of seventeen would be expected to have a negligible effect on total costs of transportation, because of the long distances involved for all sources. The
meat was then distributed from the two border stations using the Public Utilities Commission rates and mileage distances from those points to the consuming regions. A charge of 5 cents per 100 pounds per 50 miles travelled was placed on inshipments for the distance travelled outside the state to the two border stations. ${ }^{18}$ This assessment resulted in increasing the transportation costs per 100 pounds from Truckee and Blythe to the consuming areas by 60 cents and 95 cents, respectively.

A further assumption was made that all incoming meat would be slaughtered in plants killing at least 120 head per hour. Because data on the costs of slaughtering in other states were not available, the Los Angeles processing rate was assigned to out-of-state meat. ${ }^{10}$

For live animals the out-of-state distances were 488 miles from Twin Falls, Idaho, to Truckee, and 170 miles from Phoenix, Arizona, to Blythe. In this case, a charge of 7 cents per 100 pounds (dressed weight equivalent) for every 25 miles travelled was added to the cost of shipping from the two entry points to consuming regions. This resulted in an addition of $\$ 1.33$ for shipments from Twin Falls and 49 cents per 100 pounds for shipments from Phoenix.

Tables 4 and 5 give the transportation charges per 100 pounds, on a dressed weight basis, for live animals and carcass beef.

## PROCESSING COSTS

The long-run costs of slaughtering cattle were taken from a previous study (Logan and King, 1962). Because a detailed explanation of the cost estimation procedure is available elsewhere, only a summary explanation will be given here.

Plants under consideration were

[^14]Table
TRANSPORTATION RATES FOR (Dressed Weight.


Source: Rates are based on Minimum Rate Tariff 3-A, effective October 28, 1960, section 2, Distance Commodity Rates, page 18,
Distances to which the rates were applied in order to derive the above figures were taken from Distance Table 4 containing Regula.
those slaughtering only cattle without horizontal operations such as rendering, boning, sausage-making, or hide curing. Costs of live animals were not considered. Two major technologies of cattle slaughtering were studied: (1) the conventional single rail or bed-type system, and (2) the on-the-rail dressing system. In the long run, the conventional plants alter scale by adding or eliminating the "beds" or work areas in which the carcass is skinned, eviscerated, and the hooves removed; that is, there are one-bed plants, two-bed plants,
and so on. On-the-rail systems, however, change scale by lengthening or shortening the existing line and using more, or fewer, workers along the line.

For the most part, costs were estimated for various levels of output by synthesizing aggregate production and cost relationships. This method involves the derivation of elementary input-output relationships and then, through the use of these relationships as "building blocks," the construction of production and cost functions. ${ }^{20}$ The input-output relationships can be obtained through

[^15]SHIPPING LIVE ANIMALS
Equivalent）

|  |  |  | $\begin{aligned} & \text { 咢 } \\ & \text { ت⿹\zh26灬 } \\ & \text { Hin } \end{aligned}$ |  |  |  | $\begin{aligned} & \text { 罸 } \\ & \text { 别 } \end{aligned}$ |  |  |  |  |  |  | 荡 | \％ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |

100 pounds

tions，Mileage Distances Between Points within the State of California．Effective January 1，1952，pages 1－160．Both data sources are issued by the Public Utilities Commission of California，San Francisco．
work sampling，time studies，interviews with management personnel，or through direct use of accounting data．

Basically，the analytical model was as follows：
I．Construction of eight hypothet－ ical or model plants with specified designed capacities．${ }^{21}$
A．Determination of the physical
input requirements for labor and management and utilities， and the application of appro－ priate cost rates thereto．${ }^{22}$
B．Determination of investment requirements，first in a phys－ ical sense and then in a cost sense，through application of manufacturers＇prices．

[^16]TABLE
TRANSPORTATION RATES FOR

| Region and city | $\begin{aligned} & \text { 忍 } \\ & \text { y } \\ & \text { 可 } \end{aligned}$ | 䃾 | 咢 \＃ 4 |  |  |  | 朢 | 哭 | 号 | $\begin{aligned} & \text { 巻 } \\ & \text { 号 } \\ & \stackrel{y}{m} \end{aligned}$ | 可 | \％ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | cents per |  |  |  |  |  |  |  |  |  |  |  |
| 1．Eureka． | 0 |  |  |  |  |  |  |  |  |  |  |  |
| 2．Yreka． | 80 | 0 |  |  |  |  |  |  |  |  |  |  |
| 3．Alturas． | 86 | 58 | 0 |  |  |  |  |  |  |  |  |  |
| 4．Redding． | 62 | 46 | 57 | 0 |  |  |  |  |  |  |  |  |
| 5．Susanville． | 86 | 73 | 44 | 57 | 0 |  |  |  |  |  |  |  |
| 6．Red Bluff． | 66 | 53 | 62 | 24 | 48 | 0 |  |  |  |  |  |  |
| 7．Ukiah． | 62 | 80 | 86 | 62 | 77 | 57 | 0 |  |  |  |  |  |
| 8．Willows | 73 | 62 | 66 | 35 | 58 | 28 | 46 | 0 |  |  |  |  |
| 9．Oroville． | 77 | 62 | 69 | 40 | 53 | 33 | 53 | 26 | 0 |  |  |  |
| 10．Marysville | 80 | 66 | 73 | 44 | 58 | 37 | 48 | 30 | 22 | 0 |  |  |
| 11．Fairfield． | 77 | 77 | 83 | 58 | 69 | 51 | 44 | 43 | 43 | 35 | 0 |  |
| 12．Sacramento | 80 | 73 | 80 | 55 | 64 | 48 | 53 | 37 | 35 | 28 | 26 | 0 |
| 13．Jackson． | 90 | 83 | 86 | 64 | 73 | 60 | 62 | 51 | 48 | 43 | 40 | 33 |
| 14．Stockton． | 86 | 80 | 104 | 62 | 73 | 58 | 57 | 48 | 46 | 40 | 30 | 28 |
| 15．Oakland． | 80 | 83 | 90 | 64 | 77 | 62 | 51 | 53 | 53 | 46 | 30 | 40 |
| 16．Modesto． | 90 | 83 | 90 | 66 | 77 | 62 | 62 | 55 | 53 | 46 | 37 | 35 |
| 17．Bridgeport． | 166 | 160 | 166 | 150 | 155 | 146 | 146 | 138 | 138 | 134 | 127 | 127 |
| 18．Salinas． | 93 | 97 | 104 | 80 | 90 | 77 | 66 | 66 | 66 | 64 | 53 | 58 |
| 19．Merced． | 93 | 90 | 97 | 73 | 80 | 66 | 66 | 62 | 60 | 55 | 46 | 44 |
| 20．Madera． | 100 | 93 | 100 | 77 | 86 | 73 | 69 | 64 | 64 | 60 | 55 | 51 |
| 21．Fresno． | 104 | 97 | 104 | 80 | 90 | 77 | 73 | 66 | 66 | 64 | 58 | 57 |
| 22．Visalia． | 107 | 104 | 111 | 86 | 93 | 83 | 80 | 77 | 73 | 69 | 64 | 62 |
| 23．Bishop．．． | 155 | 150 | 155 | 130 | 141 | 127 | 127 | 118 | 118 | 114 | 107 | 107 |
| 24．San Luis Obispo． | 114 | 114 | 127 | 97 | 111 | 93 | 86 | 86 | 90 | 83 | 73 | 77 |
| 25．Bakersfield． | 118 | 111 | 122 | 93 | 104 | 90 | 90 | 83 | 83 | 80 | 73 | 73 |
| 26．Santa Barbara． | 134 | 134 | 141 | 114 | 127 | 111 | 104 | 104 | 104 | 100 | 90 | 93 |
| 27．Ventura． | 138 | 134 | 141 | 114 | 127 | 111 | 107 | 104 | 104 | 97 | 93 | 90 |
| 28．Los Angeles．． | 141 | 134 | 141 | 114 | 127 | 111 | 111 | 104 | 104 | 97 | 93 | 90 |
| 29．San Bernardino． | 146 | 141 | 150 | 122 | 134 | 118 | 114 | 111 | 107 | 104 | 100 | 97 |
| 30．Indio．．． | 155 | 150 | 155 | 134 | 146 | 130 | 130 | 122 | 122 | 114 | 111 | 107 |
| 31．San Diego． | 155 | 150 | 155 | 134 | 146 | 130 | 130 | 122 | 122 | 118 | 111 | 111 |
| 32．El Centro．． | 166 | 160 | 166 | 146 | 155 | 141 | 141 | 134 | 134 | 130 | 122 | 122 |
| 33a．North，out－of－state．． | 157 | 146 | 124 | 126 | 108 | 124 | 129 | 120 | 115 | 108 | 117 | 108 |
| 34a．South，out－of－state．． | 261 | 255 | 261 | 245 | 250 | 241 | 241 | 233 | 233 | 225 | 222 | 217 |

Source：Rates are based on Minimum Rate Tariff 2，effective September 23，1960，section 2，page 41．Distances to which the rates were applied in order to derive the above figures were taken from Distance Table 4 containing Regulations，Mileage Distances Between

C．Determination of miscellane－ ous variable costs of opera－ tion．
D．Determination of the total costs for model plants．
II．Estimation of short－run cost functions for model plants by varying the rate of output．
III．Determination of the long－run cost function by fitting an en－ velope to short－run cost points derived for the model plants．
The eight model plants include three conventional plants with designed ca－ pacities of 17,35 ，and 50 head per hour
（equivalent to one bed，two beds，and three beds，respectively）and five on－ the－rail plants capable of killing 20,40 ， 60,75 ，and 120 head per hour．The cattle had an average live weight of about 1,100 pounds and average dressed weight of between 600 and 700 pounds．

## Labor

Labor input requirements were syn－ thesized from time studies，accounting data，and interviews with management personnel obtained from actual plants in California，and from data provided by on－the－rail equipment－making con－

SHIPPING DRESSED BEEF

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| 30 | 0 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 48 | 37 | 0 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 37 | 22 | 40 | 0 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 127 | 118 | 127 | 114 | 0 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 60 | 48 | 43 | 44 | 114 | 0 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 46 | 33 | 46 | 25 | 107 | 44 | 0 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 53 | 43 | 55 | 35 | 104 | 46 | 24 | 0 |  |  |  |  |  |  |  |  |  |  |  |  |
| 57 | 46 | 58 | 40 | 100 | 51 | 30 | 21 | 0 |  |  |  |  |  |  |  |  |  |  |  |
| 64 | 55 | 64 | 48 | 93 | 58 | 40 | 33 | 26 | 0 |  |  |  |  |  |  |  |  |  |  |
| 107 | 86 | 100 | 97 | 46 | 97 | 90 | 86 | 83 | 77 | 0 |  |  |  |  |  |  |  |  |  |
| 80 | 69 | 66 | 66 | 107 | 51 | 62 | 58 | 55 | 55 | 90 | 0 |  |  |  |  |  |  |  |  |
| 73 | 64 | 73 | 62 | 83 | 64 | 55 | 48 | 44 | 35 | 66 | 53 | 0 |  |  |  |  |  |  |  |
| 97 | 86 | 83 | 83 | 97 | 66 | 80 | 77 | 69 | 66 | 80 | 44 | 57 | 0 |  |  |  |  |  |  |
| 93 | 86 | 86 | 80 | 93 | 73 | 77 | 69 | 66 | 62 | 77 | 51 | 51 | 24 | 0 |  |  |  |  |  |
| 93 | 86 | 93 | 80 | 90 | 83 | 77 | 69 | 66 | 62 | 73 | 64 | 51 | 43 | 35 | 0 |  |  |  |  |
| 97 | 90 | 100 | 86 | 86 | 90 | 83 | 77 | 73 | 69 | 69 | 69 | 58 | 55 | 48 | 33 | 0 |  |  |  |
| 111 | 104 | 111 | 97 | 97 | 100 | 93 | 86 | 86 | 80 | 80 | 83 | 69 | 66 | 62 | 48 | 37 | 0 |  |  |
| 111 | 104 | 111 | 100 | 104 | 100 | 93 | 90 | 86 | 80 | 86 | 80 | 69 | 64 | 62 | 46 | 46 | 57 | 0 |  |
| 122 | 114 | 122 | 111 | 111 | 114 | 104 | 100 | 97 | 93 | 93 | 93 | 83 | 80 | 73 | 64 | 55 | 37 | 51 | 0 |
| 117 | 118 | 124 | 122 | 206 | 137 | 126 | 133 | 137 | 143 | 187 | 157 | 150 | 174 | 171 | 171 | 178 | 190 | 190 | 201 |
| 222 | 213 | 222 | 206 | 206 | 209 | 202 | 199 | 195 | 188 | 188 | 192 | 178 | 175 | 172 | 159 | 153 | 138 | 164 | 152 |

Points within the State of California．Effective January 1，1952，pages 1－160．Both data sources are issued by the Public Utilities Com－ mission of California，San Francisco．
cerns．Time studies of various kill floor operations were taken to estimate the capacity in head per hour for various sizes of crews．For all operations in a one－bed plant，and for all operations requiring just one worker in any scale of plant，the capacity was the time（in minutes）required to handle one carcass divided into 60 minutes per hour．For operations in multiple bed－plants using crews of workers（such as the flooring operation and the removal of hide from rump and back），the crews work over all beds simultaneously as output increases． Until all beds are operated at full ca－
pacity per bed，production is some per－ centage of the capacity observed by tim－ ing one animal．

In synthesizing the entire killing crew for a particular output，the number of workers and their particular jobs are specified in such a manner as to make their respective capacities harmonious with the other laborers＇production levels in order to minimize excess capacity． Thus，in the smaller plants workers may perform a combination of tasks such as killing and skinning，removing，and washing the head．

The over－all plant capacity in conven－
tional slaughtering plants is generally determined by the speed of the "rumping and backing" operation-removing the hide from the rump and the back of the animal (the evisceration is performed at the same time). Production per bed reaches a maximum ( 17.6 head per hour per bed) with two workers doing these skinning operations simultaneously on the same animal. Thus, the plant capacities for the conventional plants were set at 17, 35, and 50 head per hour for the one-, two-, and threebed plants, respectively.

Standards for cooler, dock, and cleaning crew personnel were derived from time requirements estimated by Hammons and Miller (1961) in a study of Texas slaughter plants. With each plant assumed to load out one day's kill every day, cooler workers in small plants also perform the loading jobs. Such workers might also clean equipment, and wash and oil the beef trolleys.

Maintenance and yard crew requirements were based on a sample of 14 plants in California. Generally, there were definite groupings of plants with similar outputs and having the same number of yard and maintenance men. Modal values of these groupings were used to select the crew sizes.

The number of office, buying, and selling personnel specified for the model plants was based on observations of the
eight plants studied in detail, plus interviews with management personnel in the industry. In the smaller plants ownermanagers perform some of the bookkeeping tasks, while in larger plants the office assignments become more specialized. The synthesized office force was developed on the basis that as daily kill increased, the various accounting and bookkeeping operations would require more specialized attention.

Determining the management personnel for packing plants is a complex task since in many independent plants, and particularly in small plants, managers and owners are the same. Despite this complexity, interviews with slaugh-ter-plant officials and observations of the division of management responsibilities in the actual plants studied exhibited a hierarchy of command calling for a general manager, senior buyer, and sales manager. These officials handle plant operations, buying, and selling, respectively.

When output reached 40 head hourly in the eight plants surveyed, a plant superintendent was added to manage the plant operations and serve as liaison between the production area and front office. At 60 head per hour, an assistant plant superintendent was added. Table 6 shows the number of employees in the various departments of the eight model slaughter plants.

Table 6
LABOR REQUIREMENTS FOR EIGHT MODEL PLANTS

| Operation | Number of employees required for plants with hourly kill rates of: |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 17 head | 20 head | 35 head | 40 head | 50 head | 60 head | 75 head | 120 head |
| Kill crew. | 13 | 17 | 25 | 32 | 37 | 44 | 53 | 83 |
| Cooler, dock crew... | 4 | 5 | 8 | 9 | 10 | 12 | 12 | 17 |
| Maintenance crew.. | 1 | 2 | 3 | 5 | 5 | 6 | 7 | 10 |
| Yard crew. | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |
| Clean-up crew | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 5* |
| Office. | 2 | 3 | 4 | 5 | 6 | 7 | 7 | 13 |
| Buyers. | 1 | 1 | 2 | 2 | 3 | 5 | 5 | 7 |
| Sellers. . | 2 | 2 | 3 | 3 | 4 | 5 | 6 | 9 |
| Management.......... | 1 | 1 | 3 | 4 | 4 | 5 | 5 | 5 |
| TOTAL | 26 | 33 | 51 | 64 | 74 | 89 | 101 | 152 |

[^17]Table 7
COSTS OF OPERATION AT DESIGNED CAPACITIES FOR EIGHT MODEL PLANTS*

| Operation | Costs for plants with hourly kill rates of: |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Conventional dressing |  |  | On-the-rail dressing |  |  |  |  |
|  | 17 head | 35 head | 50 head | 20 head | 40 head | 60 head | 75 head | 120 head |
|  | dollars |  |  |  |  |  |  |  |
| Labor: |  |  |  |  |  |  |  |  |
| Union. | 134,497 | 258,557 | 376,511 | 173,435 | 328,753 | 436,978 | 506,029 | 766,881 |
| Salaried | 61,800 | 124,340 | 179,640 | 71,840 | 145,140 | 229,040 | 239,440 | 346,780 |
| Investment: |  |  |  |  |  |  |  |  |
| Interest. | 8,049 | 11,574 | 15,749 | 9,671 | 14,809 | 22,355 | 26,062 | 38,985 |
| Insurance. | 1,254 | 1,772 | 2,405 | 1,515 | 2,282 | 3,458 | 4,022 | 6,004 |
| Taxes. | 3,801 | 5,532 | 7,494 | 4,736 | 7,183 | 10,930 | 12,715 | 18,958 |
| Depreciation. | 13,192 | 19.548 | 26,239 | 16,066 | 24,524 | 37,162 | 43,185 | 63,108 |
| Utilities: |  |  |  |  |  |  |  |  |
| Electricity . | 10,232 | 16,852 | 22,399 | 11,362 | 18,725 | 26,074 | 31,576 | 48,147 |
| Water. | 1,612 | 3,258 | 4,463 | 1,970 | 3,695 | 5,395 | 6,458 | 9,665 |
| Gas. | 1,226 | 1,983 | 2,415 | 1,486 | 2,204 | 2,806 | 3,288 | 4,975 |
| Miscellaneous: |  |  |  |  |  |  |  |  |
| Repair, maintenance. | 10,849 | 22,382 | 32,035 | 21,814 | 25,628 | 38,443 | 48,010 | 76,885 |
| Killing supplies...... | 10,659 | 14,537 | 17,784 | 11,320 | 15,629 | 19,938 | 23,156 | 32,866 |
| Office supplies. . | 5,130 | 6,797 | 8,192 | 5,414 | 7,266 | 9,118 | 10,502 | 14,675 |
| Taxes, licenses. | 11,654 | 21,826 | 30,340 | 13,387 | 24,689 | 35,991 | 44,430 | 69,898 |
| Telephone..... | 9,736 | 18,887 | 26,547 | 11,295 | 21,463 | 31,631 | 39,223 | 62,136 |
| Delivery... | 14,818 | 22,030 | 28,067 | 16,046 | 24,060 | 32,074 | 38,057 | 56,114 |
| Feed, corral. | 3,200 | 6,602 | 9,450 | 3,780 | 7,560 | 11,340 | 14,162 | 22,680 |
| Buying..... | 1,600 | 3,301 | 4,725 | 1,890 | 3,780 | 5,670 | -7,081 | 11,340 |
| TOTAL COST. . | 303,309 | 559,778 | 794,455 | 368,027 | 677,390 | 958,403 | 1,097,396 | 1,650,097 |
| Cost per head. | 9.48 | 8.48 | 8.41 | 9.74 | 8.96 | 8.45 | 7.75 | 7.28 |

[^18]In estimating the labor costs for the model plants, the wage scale for the conventional plants was based on the Los Angeles union contract between the independent meat packing companies and the butchers' union; for the on-therail plants, the wage brackets were taken from the contract between Swift and Company and the United Packinghouse Workers of America. ${ }^{23}$ The total estimated cost for union or hourly workers is found in table 7.

The salary rates applied to office, buying, and selling employees were selected from pay scales reported by slaughter-
ing plants in the Los Angeles area. Salaries for comparable office jobs in the plants studied were generally the same without regard to size of plant, so no adjustment was made for scale of plant.

In many cases, combined ownershipmanagement distorts the salary paid executives, as the payment allocated to management may represent profits and salaries combined. The salary rates applied in this study were based on salary figures given by a packing industry analyst for executives in similar types of work and in some non-independent or chain meat packing plants.

[^19]
## Investment

Varying ages of plants, inflationary and/or depressionary trends, plus diversity among firms in accounting policies, make the use of accounting data in determining the investment in buildings, land, and equipment unreliable. Therefore, the physical requirements of these items were synthesized and the appropriate costs were applied to the various inputs. From the investment figures, annual charges for depreciation, insurance, taxes, and interest were calculated as additional "building blocks" in the over-all cost structure.

## Buildings and Improvements

Building costs were derived by use of architectural drawings of various sizes of killing floors and office space, manufacturers' specifications for boiler and refrigeration units, and, in the case of basement, storage, and employee dressing areas, extrapolation from a detailed drawing of a two-bed plant. Coolers were "constructed" by specifying 2 feet of rail space per head of cattle and then spacing rails 3 feet apart in the chill cooler and 2.5 feet apart in the storage cooler.

Construction cost rates were obtained from a meat industry report (Rothra,
1961) and were verified for the Los Angeles area by architects in that region. Table 8 shows the building investment costs.

## Corrals

Corrals were specified to have capacities approximately 2.5 times the number of animals killed daily. Pens were concrete, 10 feet by 20 feet, designed to hold 11 head. Cost rates used in deriving corral construction costs were $\$ 2.75$ per square foot for waterproof concrete, $\$ 2.25$ per linear foot for 6 -foot wood fences, and $\$ 45.00$ for the 10 -foot swinging gates.

Linear regression functions were derived relating the synthesized square footage of corral space and the footage of fencing required, respectively, to the total number of cattle in the pens. The equations were:

$$
\begin{aligned}
& C=758.873+22.702 X_{1} \\
& r^{2}=0.99
\end{aligned}
$$

where $C=$ total square footage of corrals
$X_{1}=$ total number of cattle in pens
and $\quad F=172.462+2.484 X_{1}$
$r^{2}=0.99$
where $F=$ total footage of fencing
$X_{1}=$ previously defined.

Table 8
INVESTMENT REQUIREMENTS FOR EIGHT MODEL PLANTS

| Operation | Investment requirements for plants with hourly kill rates of : |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Conventional dressing |  |  | On-the-rail dressing |  |  |  |  |
|  | 17 head | 35 head | 50 head | 20 head | 40 head | 60 head | 75 head | 120 head |
|  | dollars |  |  |  |  |  |  |  |
| Building. | 149,845 | 185,944 | 258,327 | 152,214 | 225,214 | 325,020 | 375,486 | 563,594 |
| Corrals. | 24,899 | 50,274 | 69,339 | 29,632 | 56,629 | 83,649 | 104,292 | 164,939 |
| Land. | 8,660 | 15,690 | 21,950 | 9,712 | 18,590 | 26,804 | 32,202 | 49,382 |
| Equipment*. | 69,078 | 110,405 | 143,066 | 113,244 | 163,219 | 264,597 | 303,634 | 443,074 |
| TOTAL | 252,482 | 362,313 | 492,682 | 304,802 | 463,652 | 700,070 | 815,614 | 1,220,989 |

[^20]
## Land

No definite policy with regard to the area of land owned was found in the actual plants surveyed. The amount of land set aside for future expansion, of course, may depend on the development of the area immediately surrounding the plant. For this analysis, land needs were defined as only the area required for buildings and corrals. A cost rate of $\$ 0.50$ per square foot of land, based on reports by the Los Angeles County tax office, was used to calculate land investment (table 8).

## Equipment

The physical equipment requirements for conventional plants were based on observations and itemized equipment breakdowns from the seven plants studied in detail. For the on-the-rail plants, the equipment needs were taken from specifications of actual plants obtained from the Allbright-Nell Co., Chicago equipment manufacturer. Manufacturers' prices, freight charges, and a 4 per cent sales tax were used to estimate costs of equipment.

On-the-rail equipment was assumed to originate in Chicago and move to Los Angeles, and shipments for conventional plants were assumed to go from San Francisco to Los Angeles. Items identical to both systems were assumed to originate in San Francisco in order to take advantage of lower transportation costs.

Specifications and costs of pumps, boilers, and refrigeration equipment were obtained from California concerns manufacturing these items.

## Annual Costs of Investment

Five major cost items arise from the investment of the firm: depreciation, insurance, property taxes, interest, and repair and maintenance. (The latter will be considered elsewhere.)

Following a procedure common to many California slaughter houses, a straight-line depreciation policy was
followed using the total installed cost of goods (including sales cost, installation cost, freight cost, and sales tax) and then subtracting the salvage value of the item from the installed cost. Uselives for specific items were taken from firm accounting records with modal values used where possible.

On a per-head basis, depreciation costs show economies of scale for the conventional and on-the-rail plants. Such costs decrease from 18 cents for the one-bed plant to 13 cents for the two- and three-bed plants. For on-therail plants, the depreciation costs drop from 22 cents per head for 20 head per hour to 15 cents for the 120 head per hour. Per head cost, however, jumps to 18 cents from 16 cents, as the kill rate goes from 40 to 60 head hourly. This apparent diseconomy is attributable to the change in equipment requirements. The larger plant uses a moving top viscera table and more conveyors which eliminate hauling the viscera in hand trucks. (Larger federally inspected on-the-rail plants are required by the government to utilize a moving top table to facilitate viscera inspection.)

The personal property tax rate for Los Angeles County for 1960-61 (8.0964) was applied to the synthesized plants. Application of the tax rate to the full assessed valuation would not allow for the decrease in actual market value because of depreciation. Therefore, a tax rate equal to one-half the regular rate was applied to the assessed value of the depreciable property to show the average charges paid for property taxes over the life of the equipment and buildings. The full tax rate was applied to land investment and to the salvage value of the equipment since these values do not depreciate.

A similar procedure was followed with insurance and interest on investment, but charges were applied to the full market value rather than to the assessed value. Base rates used were 1 per cent of market value for insurance and 6 per cent per year interest.

## Utilities

Functions were derived regressing annual consumption of water and electricity in physical units on the number of animals killed. A least squares regression linear function relating electrical consumption of plants to the size of slaughter yielded the following coefficients:

$$
\begin{aligned}
E & =157,183.647+15.448 X \\
r^{2} & =0.8294
\end{aligned}
$$

where $\bar{E}=$ yearly consumption of electricity in kilowatt hours and $X=$ annual slaughter in number of head.

Usage of water under the specifications of operation for the model plants is mainly for cleaning or washing opera-tions-that is, cleaning of the dressed carcass, washing the head, the tripe, scalding the tripe, and cleaning the floor and equipment. In many of the plants studied, additional operations such as rendering or sausage-making made the data regarding water usage unacceptable for statistical analysis since these operations require large amounts of water. Data from four plants were used to estimate a linear function passing through the mean and the origin. ${ }^{24}$ The function was $W=0.362 X$, where $W=$ annual water consumption in 100 cubic feet and $X=$ yearly slaughter in number of head.

Since the major fuel item in slaughtering plants is for heating boilers, gas requirements were estimated from the amount of fuel needed to operate the boilers for 8 hours daily. The resulting figures were then adjusted to a monthly basis.

The cost rates applied to the three utility requirements were taken from
rates charged by the various companies in the Los Angeles area. ${ }^{25}$ Consumption was adjusted to a monthly basis for application of the cost schedules and was then readjusted to the annual cost basis.

## Miscellaneous Costs

There are several other inputs used by slaughtering plants whose costs are relatively small individually; when considered collectively, however, they become important cost items. These catagories include nonlabor costs of repair and maintenance, killing supplies, office supplies, taxes and licenses, telephone calls, delivery and selling supplies, feed and corral items, and buying suppliesitems having little basis on which they can be accurately synthesized. In these cases, accounting data obtained from the slaughtering plants surveyed were used to derive statistical cost functions. Where data did not lend themselves to regression analysis because of nonhomogeneous nature, use was made of a mean or modal figure.

In all cases, the cost variable was considered to be a function of annual slaughter in number of head; table 9 gives the resulting coefficients.

## Total Costs

The costs for the various inputs were combined into total costs for the model plants (table 7). For the three conventional bed-type plants, output was varied for each plant in order to derive short-run cost functions. For the short run, it was assumed that only union labor and miscellaneous costs were variable with depreciation, insurance, taxes, interest, and costs of utilities remaining constant. ${ }^{23}$ The indivisibility of labor

[^21]Table 9
ESTIMATED FUNCTIONAL COEFFICIENTS BETWEEN VARIOUS COSTS AND YEARLY OUTPUT

| $\underset{\text { variable }}{\text { Dependent }}$ | Coefficient of determination, $r^{2}$ | Constant term | Regression coefficients |
| :---: | :---: | :---: | :---: |
|  |  | dollars | dollars per head |
| Killing costs..... | 0.26 | 7,010.745 | $\begin{gathered} 0.114 \\ (0.087) \dagger \end{gathered}$ |
| Office costs...... | 0.78 | 3,561.957 | $\begin{gathered} 0.049 \\ (0.013) \end{gathered}$ |
| Taxes and licenses. | 0.98 | 2,084.840 | $\begin{gathered} 0.299 \\ (0.028) \end{gathered}$ |
| Telephone | 0.89 | 1,126.435 | $\begin{gathered} 0.269 \\ (0.048) \end{gathered}$ |
| Delivery ........ | 0.61 | 8,032.860 | $\begin{gathered} 0.212 \\ (0.085) \end{gathered}$ |
| Repair and maintenance. | * | 0 | 0.339 |
| Feed and corral. | * | 0 | 0.100 |
| Buying costs.... | * | 0 | 0.050 |

* These items were not estimated.
$\dagger$ Standard errors of the coefficients are in parentheses. Source: Logan, Samuel H, and Gordon A. King, Economies of Scale in Beef Slaughter Plants, Giannini Foundation Rese
inputs resulted in a cost function with a step nature, and average short-run cost functions which are discontinuous (fig. 7 ). The labor force is constant for small shifts in output, but it varies over the entire range of output for a particular plant. Therefore, within the various production steps for a given plant, the only variable cost becomes miscellaneous costs.

Since data for on-the-rail plants were not obtained for varying levels of output, short-run costs were not estimated. The points associated with average costs for the respective plants operated at capacity were specified to lie on the longrun cost function for their respective technologies.

Because of the indivisible nature of the killing floor over large ranges of output in conventional plants and of the labor crew for smaller production ranges, the long-run average cost func-
tion will be discontinuous. The envelope curve shown in figure 7 illustrates the nature of the actual cost curve but does not indicate actual discontinuities.

## Cost Adjustments

Slaughtering costs derived for the Los Angeles area were adjusted to allow for regional differences in tax rates, utility schedules, and wage rates. Costs were adjusted for the on-the-rail plant killing 120 head hourly ( 142 million pounds dressed weight annually), and the regional cost differentials were specified to remain constant over all sizes of plants. (A similar adjustment for one-bed plants showed little variation from the differential calculated for the large on-the-rail plant.)

Tax rates vary considerably within a county from one district to another, depending on outstanding improvement bond charges, etc. Thus, any adjustment of the Los Angeles charge should be considered as, at best, an approximation. The base tax rate was obtained for each county in which a point of origin was located (Cranston, 1960). City tax rates where applicable, were added to the county rate (Cranston, 1961). The ratio of the combined county and city rates of other regions to that of Los Angeles then served as an index by which the tax charges in Los Angeles were adjusted to comply with the tax rates in the other slaughtering areas. Such a procedure assumes uniform assessment ratios over the state (table 10).

Utility rates for water, gas, and electricity by point of origin were obtained from the California Public Utilities Commission and from the individual companies. Rate differences for industrial usage of gas and electricity for the most part split geographically between northern and southern California. For electricity, the southern counties of Los Angeles, Orange, Imperial, San Diego, Riverside, San Bernardino, Santa Barbara, Ventura, Kings, and Tulare were specified to be serviced by Southern California Edison at the same cost as


Fig. 7. Envelope curve to the long-run average cost points for the eight synthesized plants (inset in upper right shows envelope to the three short-run average cost functions for the conventional plants).
for Los Angeles County. Most of the remainder of the state was supplied by the Pacific Gas and Electric Company. The rates charged by these two concerns were assumed to be indicative of the cost of electricity in the few counties not included in their service areas. ${ }^{27}$

Natural gas service similarly divided into two major service areas with Pacific Gas and Electric again supplying the northern part of the state and Southern California Gas Company supplying the remainder at the same cost as for Los Angeles. ${ }^{28}$ The same area boundary used for electricity was also applicable to the natural gas service.

Although electrical and gas service
divide into two regions, the water rates from county to county show no such pattern. Much of the water service in the separate cities is supplied by municipal water districts or small individual companies. Therefore, individual water rates were obtained for the origin cities, and charges for this utility were adjusted accordingly.

In considering possible alterations in labor costs by area, union contracts for slaughterhouse workers were obtained from 17 local unions over the state. Many of the wage scales for neighboring regions were the same, or exhibited only minute differences. Five distinct differences in wage scales were evident, how-

[^22]Table 10
AN INDEX OF VARIOUS COST COMPONENTS OF SLAUGHTERING IN SUPPLY REGIONS AS A PER CENT OF LOS ANGELES COSTS

| Central point | Water | Gas | Electricity | Taxes | Labor | Total costs |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | per cent $($ Los Angeles $=100)$ |  |  |  |  |  |
| Eureka | 128 | 110 | 132 | 132 | 105 | 105 |
| Yreka. | 127 | 110 | 132 | 96 | 105 | 105 |
| Alturas. | 128 | 110 | 132 | 93 | 105 | 105 |
| Redding. | 128 | 110 | 132 | 75 | 105 | 104 |
| Susanville. | 43 | 110 | 132 | 89 | 105 | 104 |
| Red Bluff | 60 | 110 | 132 | 100 | 105 | 104 |
| Ukiah | 60 | 110 | 132 | 104 | 105 | 104 |
| Willows. | 148 | 110 | 132 | 88 | 105 | 105 |
| Oroville | 105 | 110 | 132 | 106 | 105 | 105 |
| Marysville. | 75 | 110 | 132 | 90 | 105 | 104 |
| Fairfield | 128 | 110 | 132 | 97 | 108 | 106 |
| Sacramento. | 73 | 110 | 132 | 94 | 105 | 104 |
| Jackson | 112 | 110 | 132 | 60 | 105 | 104 |
| Stockton | 81 | 110 | 132 | 133 | 105 | 105 |
| Oakland | 98 | 110 | 132 | 130 | 108 | 107 |
| Modesto. | 79 | 110 | 132 | 106 | 106 | 105 |
| Bridgeport. | 82 | 110 | 132 | 32 | 106 | 104 |
| Salinas. | 88 | 110 | 132 | 83 | 101 | 102 |
| Merced | 65 | 110 | 132 | 92 | 101 | 102 |
| Madera. | 54 | 110 | 132 | 97 | 101 | 102 |
| Fresno. | 85 | 110 | 132 | 114 | 101 | 102 |
| Visalia | 70 | 100 | 100 | 94 | 101 | 101 |
| Bishop. | 87 | 110 | 132 | 59 | 101 | 101 |
| San Luis Obispo. | 128 | 110 | 132 | 92 | 100 | 101 |
| Bakersfield. | 69 | 110 | 132 | 98 | 100 | 101 |
| Santa Barbara. | 138 | 100 | 100 | 75 | 100 | 100 |
| Ventura. | 126 | 100 | 100 | 88 | 100 | 100 |
| Los Angeles. | 100 | 100 | 100 | 100 | 100 | 100 |
| San Bernardino. | 102 | 100 | 100 | 76 | 100 | 100 |
| Indio. | 60 | 100 | 100 | 72 | 100 | 99 |
| San Diego. | 193 | 100 | 100 | 98 | 100 | 100 |
| El Centro. | 80 | 100 | 100 | 92 | 100 | 100 |

Source: Based on utility and tax rates and union contracts prevailing in each area. Adjustment is for a plant killing 120 head hourly.
ever, so the state was divided into five regions (fig. 8) with the unions in a given region having the same wage scale. The wage scales for these five areas were then substituted for the Los Angeles wage rate in the killing crew cost analysis. An index was calculated with the total killing floor cost of each region expressed as a percentage of Los Angeles costs. This index was subsequently used to adjust the total union labor cost (including the nonkill floor personnel) in the other four regions. Salaries of nonunion personnel for the entire state were assumed to be the same as those used in the previous cost analysis.

Figure 9 shows the long-run average cost function-adjusted to millions of
pounds of carcass beef per year, and figure 10 illustrates long-run total costs for processing cattle in terms of millions of pounds of carcass beef per year. Both functions are applicable to the Los Angeles region. The general nature of the cost function is shown by the line drawn through the points representing both on-the-rail and bed-type plants. This linear total cost function approximates the function envisaged in the theoretical model. Although use has been made (for purposes of illustration) of a continuous line, the discontinuous method of replicating kill floors for expansion under the bed system, will result in an actual function over the range for bed plants which is a step function. These


Fig. 8. Areas of the state having the same union wage scales.
departures from the smooth, continuous curves of conventional theory, therefore, require comparison of total cost alternatives under the spatial equilibrium model as described in the theoret-
ical section, rather than equilibration of marginal cost functions. In addition, the function shown in figure 9 does not include assembly or distribution costs which will also vary for each region.

## LOCATION AND SIZE OF PROCESSING PLANTS

## MODEL RESULTS

The problem was set up in the transhipment framework described on pages

151 to 154 . In this case, there were 34 points of origin for live animals (including the 32 in-state regions shown in


Fig. 9. Slaughter cost for beef cattle in Los Angeles, based on long-run average cost curve.
table 12, and 2 out-of-state locations) and 32 demand centers for the final product. In the transhipment model each demand center becomes a potential supplier of dressed beef by slaughtering more than its own demand; therefore, the supply centers for dressed beef were the same 32 in-state points and the 2 different out-of-state centers for final product shipment. Similarly, the supply sources of live animals also become demand centers for live animals in order to permit transhipment through intermediate points if such a procedure results in lower cost than direct point-topoint shipment.

Consequently, the transhipment problem utilized 68 sources with 66 destina-
tions; or in terms of a cost matrix, the dimensions were 68 rows by 66 columns.
An artificial constant of 1,000 million pounds was added to each column and row. Computer restrictions prevented use of a larger constant, which would have permitted the slaughtering of the entire supply of animals at any one point; in view of the sizable transportation costs which would be incurred in such a case, however, the probability of altering the final solution by using 1,000 million pounds appeared to be negligible. Indeed, in the final solution no one point was in danger of passing the 1,000 million-pound slaughter mark.

In the initial solution of the transhipment problem, the slaughtering cost


Fig. 10. Total annual cost of slaughtering cattle for Los Angeles area with plants operating at designed capacity, one shift per day.
for each region was set at the lowest point on the long-run average cost func-tion- $\$ 1.16$ per 100 pounds of dressed beef for Los Angeles. Regional differences in processing costs were included in the program.

This particular version of the problem, then, represents a situation in which processing costs per unit of output are constant regardless of size of plant. In this event, the solution to the transhipment problem is the optimum least-cost solution. The regional varia-
tions in processing costs were not of sufficient size to warrant combined slaughtering between any of the 32 regions. As a result, the solution indicated slaughtering in all regions with plants varying in size from one million pounds annual production (region 17) to multiple plant requirements with total output of 420 million pounds annually (region 28, Los Angeles).

Surplus production regions shipped their excess supplies to deficit regions either as live animals or dressed beef

Table 11
SLAUGHTERING AND SHIPMENT PATTERNS INDICATED BY FIRST SOLUTION

depending on the combined effect of regional cost differences and transportation differentials between live animals and dressed beef shipments (table 11).

When cost differences resulting from scale of plant are interjected into the model, however, the problem becomes more complex. In setting up the second iteration of the problem, the processing costs were changed to conform with either the supply of live animals within the region or the slaughter level indicated by the first solution, whichever resulted in lower cost. The effect of this change was to greatly increase slaughtering cost per 100 pounds in the regions where supplies were small (below 30 million pounds per year). Table

Table 12
SLAUGHTERING COSTS PER 100 POUNDS OF CARCASS BEEF USED IN THE THREE TRANSHIPMENT PROBLEMS

| Region and city | First solution | Second solution | Third solution |
| :---: | :---: | :---: | :---: |
|  | cents |  |  |
| 1. Eureka | 122 | 316 | 316 |
| 2. Yreka. | 122 | 251 | 365 |
| 3. Alturas | 122 | 286 | 286 |
| 4. Redding | 121 | 285 | 285 |
| 5. Susanville. | 121 | 364 | 364 |
| 6. Red Bluff | 121 | 250 | 130 |
| 7. Ukiah. | 121 | 285 | 285 |
| 8. Willows | 122 | 191 | 191 |
| 9. Oroville | 122 | 226 | 226 |
| 10. Marysville | 121 | 165 | 165 |
| 11. Fairfield | 124 | 159 | 159 |
| 12. Sacramento. | 121 | 132 | 132 |
| 13. Jackson. | 121 | 364 | 364 |
| 14. Stockton | 122 | 145 | 148 |
| 15. Oakland | 124 | 126 | 124 |
| 16. Modesto . | 122 | 149 | 149 |
| 17. Bridgeport. | 121 | 364 | 364 |
| 18. Salinas | 118 | 143 | 145 |
| 19. Merced | 118 | 137 | 209 |
| 20. Madera | 118 | 142 | 361 |
| 21. Fresno. | 119 | 124 | 121 |
| 22. Visalia. | 117 | 120 | 120 |
| 23. Bishop. | 118 | 361 | 361 |
| 24. San Luis Obispo. | 117 | 178 | 231 |
| 25. Bakersfield . | 117 | 117 | 117 |
| 26. Santa Barbara. | 116 | 150 | 151 |
| 27. Ventura. | 116 | 140 | 146 |
| 28. Los Angeles..... | 116 | 116 | 116 |
| 29. San Bernardino.. | 116 | 129 | 230 |
| 30. Indio . | 116 | 124 | 138 |
| 31. San Diego. | 117 | 360 | 360 |
| 32. El Centro. | 116 | 116 | 116 |

12 gives the average costs of slaughtering used in the transhipment problems.

With the higher processing costs, it becomes feasible to ship animals from high slaughtering cost areas to regions with larger plants and lower costs. The solution to the second iteration indicated that slaughtering should be done in 17 regions rather than in all 32 areas. Production levels ranged from 4 million pounds to 485 million pounds. Most of the northern regions of the state shipped their animals to the Bay area, region 15, and received their dressed beef from out-of-state shipments of dressed beef.

At this point, the problem of coping with economies of scale in the transhipment (or any other linear programming type model) becomes evident. In the program solutions, shipments of live animals are made on a cost minimization basis for the costs entered in the program format. However, if live animal shipments are selected because of lower transportation cost to the destination and/or lower processing costs at the destination, the effect on slaughtering costs of reduced supply for slaughtering in the shipping region must be considered.

For example, in the second iteration, region 25 has a supply of 145 million pounds of dressed beef equivalent and a local demand of only 35 million pounds. The solution (table 13) indicates that region 25 sends 110 million pounds as live animals to region 28 . The slaughtering cost in region 25 for 145 million pounds is $\$ 1.17$ per hundredweight; the cost in region 28 is $\$ 1.16$. The transportation costs for meat and live animals on a dressed weight equivalent are identical-51 cents per 100 pounds. As a result, it is cheaper by 1 cent per 100 pounds to ship live animals and slaughter them at the destination rather than ship meat. However, reduced supply in region 25 would raise average costs of processing to $\$ 1.38$. Thus, the savings, by shipping live animals, of $\$ 11,000$ is more than offset by the increased cost - $\$ 73,500$-of slaughtering the remaining 35 million pounds.

Table 13
SLAUGHTERING AND SHIPMENT PATTERNS INDICATED BY SECOND SOLUTION


In some cases, this problem can be solved by inserting the increased costs in the program and running a new iteration. If costs are greatly increased, the reduced slaughtering operation may be economically combined with that of another region at a new lower total cost. This was true, for instance, with regard to regions 19 and 20 . The second solution indicated that these regions should ship live animals of 29 million and 27 million pounds dressed beef equivalent, respectively, to region 15 while slaughtering 12 million and 5 million pounds. As a result, processing costs in region 19 went from $\$ 1.37$ to $\$ 2.09$, while those of region 20 rose from $\$ 1.42$ to $\$ 3.61$. These higher costs were used in the third iteration of the problem. The new solution completely eliminated slaughtering in these two regions by having them ship all their animals to region 15 while being supplied from region 21.

Another problem in programs using linear cost functions is the possible combination of several high-cost plants into one large plant with lower costs. In the second iteration many of the northern regions shipped their live animals to the Bay area because their slaughtering costs were relatively too high. It is possible, however, that these regions could slaughter their combined live animal supply at a central point and in this manner reduce total costs of the program.

For the third iteration, therefore, costs were reduced for processing in one of the northern regions having available supplies in surrounding regions. The result was the establishment of a plant in that area in the third solution. The third iteration (table 14) indicated slaughtering plants in 12 of the 32 regions with annual production ranging from 2 million pounds to 542 million pounds. Because of the problems mentioned above, a budgeting procedure was performed on the regions whose shipments of live animals would result in substantially higher slaughter costs in those areas, and whose plants could
not be economically combined with those of another region. As a result, region 25 was specified to ship meat rather than live animals to region 28 , and region 14 was similarly specified to ship meat to the Bay area, thus raising its slaughter rate from 2 to 34 million pounds annually.

Of the twelve regions with slaughtering plants, six slaughtered less than or equal to the amount of their own demand. The remaining six areas received animals "from themselves," or from other areas, and transhipped the dressed beef to nonslaughtering or defi-cit-slaughtering regions.

While the adjusted third iteration is the least-cost solution, a circularity in the iterative procedure was found at this point. The processing cost of the Red Bluff plant used in the third iteration was $\$ 1.30$ per hundredweight. However, the supply of live animals received at that point in the solution was less than the quantity needed for a plant with costs of $\$ 1.30$. With a supply of about 37 million pounds (dressed weight equivalent), the processing cost would be $\$ 1.42$ per 100 pounds.

If a cost level of $\$ 1.42$ is utilized in the program, then Susanville (region 5) will send animals to the Bay area (region 15) and dressed beef for Oroville (region 9) and part of Eureka (region 1) will be supplied by out-of-state shipments. This again would cause processing costs to increase at Red Bluff because of the decrease in scale of operation. This pattern is followed until the plant at Red Bluff is eliminated; all supplying points send their live animals to the Bay area, and points formerly supplied with dressed beef from Red Bluff now receive shipments from out of state. However, this final solution results in a higher total cost (by about $\$ 44,000)$ than the adjusted third iteration because of increased cost of shipping live animals. Therefore, the adjusted third iteration is selected as the approximate optimum location and shipment pattern because of the lower

Table 14
SLAUGHTERING AND SHIPMENT PATTERNS INDICATED BY THE THIRD SOLUTION*


[^23]cost. (This problem will also be discussed in respect to prices for live animals and dressed beef.)

The total program costs for each of the solutions, using costs associated with the indicated slaughter level in each program, were as follows:

First iteration (32
slaughtering
points) . .......\$33,602,000
Second iteration (17
slaughtering
points) ....... 32,756,000
Third iteration (12
slaughtering
points) ........ 32,374,000
Third iteration with
budgeted
adjustment .... 32,266,000
Final solution (11
slaughtering points
and same budgeted
adjustments as
third iteration) . . $32,310,000$

## PRICE DIFFERENTIALS

The solution to the programming problem which minimizes total costs is also the dual solution which maximizes total revenue for producers, given the restraint that the revenue of the shipment cannot exceed the processing cost plus transportation charges. We have also pointed out that in a competitive framework the equilibrium prices among regions will differ only by the costs of transportation. Therefore it is possible, using the final solution to the transhipment problem and the transportation costs, to derive price differentials for dressed beef and then the imputed prices for the live cattle.

By setting the price in some base region, it is possible to calculate the prices in other regions. In this case, the base region was Los Angeles (region 28). The prices of dressed beef in regions which supply meat to Los Angeles would be the Los Angeles price minus the cost of transportation from the surplus region to Los Angeles. In algebraic terms:
$P_{b i}=P_{b 28}-t_{i 28}$
where $\quad P_{b i}=$ price of beef per 100 pounds in region $i$
$P_{b 28}=$ price of beef per 100 pounds in region 28
$t_{i 28}=$ cost per 100 pounds of shipping beef from region $i$ to region 28.

If the price in the base region is set at zero, then the results from the above equation are the price differentials between other regions and that of the base region.

Through the various combinations of shipments of dressed beef, the price differentials for all but three regions can be calculated. These three regions (Sacramento, Santa Barbara, and Ventura) ship only live animals or receive only live animals without shipping either meat or animals. However, the prices of dressed beef can be derived by use of the imputed prices of the live cattle, as shown on page 182.

Table 15 gives the price differentials for dressed beef from the final adjusted solution; the equilibrium beef prices for various regions are also shown.

In a similar manner, the imputed prices for cattle can be calculated. In this case, the prices for cattle equal the price of dressed beef less the cost of slaughtering in the region where they are processed plus an adjustment for the transportation costs. In other words:

$$
P_{c i j}=P_{b 28}+D_{j 28}-c_{j}-t_{i j}
$$

where $\quad P_{c i j}=$ price of cattle per 100 pounds shipped from region $i$ to region $j$ (in dressed weight equivalent)
$P_{b 28}=$ price of dressed beef per 100 pounds in region 28
$D_{j 28}=$ differential in price in region $j$ and price in region 28
$c_{j}=$ cost per 100 pounds of slaughtering in region $j$
$t_{i j}=$ cost per 100 pounds of transporting animals from region $i$ to region $j$

Table 15
PRICE DIFFERENTIALS AMONG REGIONS FOR BEEF AND LIVE ANIMALS BASED ON THIRD ADJUSTED SOLUTION*

| Region and central point | Beef differential ( $\mathrm{LA}=0$ ) | Price of beef | Cattle differential $(\mathrm{LA}=0)$ | Price of cattle (dressed weight) | Price of cattle (live weight) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1. Eureka. | +0.27 | 40.27 | -1.08 | 37.76 | 23.02 |
| 2. Yreka. | +0.14 | 40.14 | -1.17 | 37.67 | 22.97 |
| 3. Alturas. | +0.03 | 40.03 | -1.36 | 37.48 | 22.85 |
| 4. Redding | -0.15 | 39.85 | -0.83 | 38.01 | 23.18 |
| 5. Susanville. | -0.13 | 39.87 | -1.13 | 37.71 | 22.99 |
| 6. Red Bluff | -0.39 | 39.61 | -0.65 | 38.19 | 23.29 |
| 7. Ukiah . | +0.08 | 40.08 | -0.56 | 38.28 | 23.34 |
| 8. Willows. | -0.11 | 39.89 | -0.57 | 38.27 | 23.34 |
| 9. Oroville | -0.06 | 39.94 | -0.57 | 38.27 | 23.34 |
| 10. Marysville | -0.13 | 39.87 | -0.51 | 38.33 | 23.37 |
| 11. Fairfield | -0.04 | 39.96 | -0.31 | 38.53 | 23.49 |
| 12. Sacramento. | -0.15 | 39.85 | -0.31 | 38.53 | 23.49 |
| 13. Jackson. | -0.04 | 39.96 | -0.53 | 38.31 | 23.36 |
| 14. Stockton. | -0.34 | 39.66 | -0.66 | 38.18 | 23.28 |
| 15. Oakland. | +0.03 | 40.03 | -0.05 | 38.79 | 23.65 |
| 16. Modesto. | -0.15 | 39.85 | -0.43 | 38.41 | 23.42 |
| 17. Bridgeport. | +0.31 | 40.31 | -1.23 | 37.61 | 22.93 |
| 18. Salinas. | -0.04 | 39.96 | -0.33 | 38.51 | 23.48 |
| 19. Merced | -0.25 | 39.75 | -0.51 | 38.33 | 23.37 |
| 20. Madera. | -0.34 | 39.66 | -0.61 | 38.23 | 23.31 |
| 21. Fresno. | -0.55 | 39.45 | -0.60 | 38.24 | 23.32 |
| 22. Visalia. | -0.62 | 39.38 | -0.66 | 38.18 | 23.28 |
| 23. Bishop . | +0.15 | 40.15 | -0.90 | 37.94 | 23.13 |
| 24. San Luis Obispo. | -0.07 | 39.93 | -0.74 | 38.10 | 23.23 |
| 25. Bakersfield. | -0.51 | 39.49 | -0.52 | 38.32 | 23.37 |
| 26. Santa Barbara. | -0.06 | 39.94 | -0.41 | 38.43 | 23.43 |
| 27. Ventura. | -0.01 | 39.99 | -0.31 | 38.53 | 23.49 |
| 28. Los Angeles. | 0.00 | 40.00 | 0.00 | 38.84 | 23.68 |
| 29. San Bernardino. | -0.09 | 39.92 | -0.28 | 38.56 | 23.51 |
| 30. Indio. | -0.27 | 39.73 | -0.48 | 38.36 | 23.39 |
| 31. San Diego. | -0.13 | 39.87 | -0.46 | 38.38 | 23.40 |
| 32. El Centro. | -0.64 | 39.36 | -0.64 | 38.20 | 23.29 |
| 33. North, out-of-state. | -1.21 | 38.79 | -2.12 | 36.72 | 22.39 |
| 34. South, out-of-state. | -1.59 | 38.41 | $-1.23$ | 37.61 | 22.93 |

${ }^{*}$ Based on a price of dressed beef of $\$ 40.00$ per 100 pounds in Los Angeles.
Source: Differentials determined by setting the price in a base region and then (using transportation rates) calculating prices in remaining regions.

Differentials can also be calculated by setting the live animal price at zero in the base region and using the following formulation:
$D_{c i j 28}=D_{b j 28}+\left[c_{28}-c_{j}\right]-t_{i j}$
where $D_{c_{i j 28}}=$ differential in price of live cattle (in dressed weight equivalent) in region $i$ which are shipped to region $j$ from the price in region 28
$D_{b j 28}=$ differential in price of beef in region $j$ and price in region 28
$c_{28}=$ cost per 100 pounds of slaughtering in region 28
$c_{j}=$ cost per 100 pounds of slaughtering in region $j$
$t_{i j}=$ cost of shipping animals from region $i$ to region $j$.

The price differentials for live animals (in dressed weight equivalents) using region 28 as the base region are given in table 15 which also gives prices of cattle in dressed weight equivalent. The latter figures were derived on the basis of the carcass beef price of $\$ 40.00$ per hundred pounds in region 28 , or a live animal price of $\$ 38.84$ ( $\$ 40.00$ less slaughtering costs of \$1.16). ${ }^{29}$

[^24]Prices of cattle on a live weight basis can be calculated by using the inverse of the figure used to adjust live cattle transportation charges to the dressed weight equivalent.

While the dual nature of the programming solution indicates prices which maximize total revenue and minimize cost for producers, the circularity of solutions explained above results in a set of live animal prices for the third adjusted solution which does not maximize returns to producers in four areas. These areas were the four regions ( 2,3 , 4 , and 5) which ship their live animals to Red Bluff. Using a slaughtering cost of $\$ 1.42$ per 100 pounds (rather than the $\$ 1.30$ used in the problem itself) the derived prices indicate producers would be better off to ship to the Bay area. However, the total cost of the program increases if the latter procedure is followed, and this cost increase means that if producers ship animals to the Bay area the price of dressed beef in those regions formerly supplied by the Red Bluff plant would rise. Prices for beef would increase 9 cents per hundred pounds in region 1, 10 cents in region 8, 11 cents in region 2, 20 cents in region 4, and 42 cents in region 6.

At this point, one might conclude that it is a matter of the welfare of the producer versus that of the consumer. However, since the objective of this study is to derive the least-cost pattern
of location and size of plant the third adjusted solution was the one used to calculate the price differentials.

Three regions did not have shipment patterns of dressed beef which allowed direct calculation of the price differentials between these areas and Los Angeles, but in these cases, shipment patterns permitted estimation of the price of live cattle in the regions. This price was simply adjusted upward by the cost of slaughter to derive the price of the finished product in the regions. A similar situation was found with respect to live animal prices for five regions which shipped only dressed beef. Here, the prices of dressed beef were adjusted downward by the cost of slaughtering to derive the prices of live animals.

The derived cattle and dressed beef prices do not show a great deal of variation over the state. The carcass beef prices range from $\$ 40.31$ in Bridgeport to $\$ 39.36$ in El Centro. (Out-of-state prices would be slightly lower.) Live animal prices (live weight basis) vary from $\$ 23.68$ in Los Angeles to $\$ 22.93$ in Alturas. On a per pound basis, the differentials would be even smaller-less than a cent a pound. It is interesting to note that the published data on slaughter cattle and dressed beef prices at a small number of points in California also indicate very little variation when considered on a per pound basis (FederalState Mkt. News Serv., 1961).

[^25]
## SUMMARY AND CONCLUSIONS

The objective of the above analysis was to estimate the optimum location and size of cattle slaughtering plants in California which would minimize the costs of shipping live animals to slaughtering points, processing them, and shipping the dressed beef to consumers. The method of analysis utilized the transhipment model of linear program-ming-a model which permitted simultaneous consideration of live animal shipments, processing, and distribution of final product.

California was divided into 32 producing and consuming regions, and two points each were designated to represent out-of-state inshipments of live animals and dressed beef, respectively. Production of feedlot cattle, dairy stock for slaughter, and culled beef herd animals was estimated by county and then combined into regional figures. A similar procedure was followed for consumption of carcass beef.

The costs of slaughtering were obtained from a previous study of economies of scale by the authors and were adjusted for regional cost differences in labor, utilities, and taxes. Transportation charges were taken from rate schedules published by the California Public Utilities Commission. (Live animal rates were adjusted to a dressed weight equivalent.)

Because of the linear nature of the programming method used, an iterative approach is required to consider economies of scale or a nonlinear average cost function. The problem was first considered with the lowest possible average cost of slaughtering for each region; then, cost rates consistent with the initial solution were substituted in the problem and another solution obtained. Three iterations were used, and the third solution was adjusted in order to take advantage of economies of scale of slaughtering in two surplus regions.

The least-cost solution with budgeted
adjustments indicated 12 slaughtering regions in the state (not including the two out-of-state origin points for carcass beef). Six of the twelve slaughtering regions remained either deficit regions or slaughtered only enough for their own supply (two of these shipped surplus in the form of animals). The other areas received animals, processed them, and shipped dressed beef elsewhere.

The volume of slaughter indicated in table 14 (after adjustment) ranged from 20 million pounds at Santa Barbara (equivalent to a one-bed plant capacity) to 432 million pounds in Los Angeles (equivalent to about three of the large on-the-rail dressing plants considered in the cost study). Such a variety in the size of plants indicates the importance of assembly and distribution costs in determining the optimum size and location of plants.

Total cost of shipments and processing (not including cost of animals) decreased from $\$ 33.6$ million in the initial iteration to $\$ 32.3$ million in the third iteration after budgeted adjustments.

Use of the transhipment model also permitted computation of equilibrium prices for both carcass beef and live animals using the price in the Los Angeles region as a base figure. Under a competitive framework, the variation in prices over the state was $\$ 0.95$ per 100 pounds for carcass beef and $\$ 0.75$ for live animals (live weight basis).

## MAJOR LIMITATIONS

One of the major limitations of most spatial equilibrium studies of this nature lies in the assumptions concerning the supply and demand functions involved. In this analysis the demand for final product within regions was specified as completely inelastic with respect to price--that is, it did not vary with price changes. Lack of data and the smallness of the regions prevented the estimation of demand functions by
statistical methods. Should the assumption of price inelasticity be incorrect the final solution and shipment pattern could be changed, depending on the nature of the individual regional functions. Not only is there the problem of price elasticity, but other variables (such as income, sociological factors, and preferences for one type or grade of beef over another) may cause a variation in the level of demand among regions that is not reflected in the use of population factors only. The magnitude of error due to this oversimplification is impossible to determine with currently available data.

The same is true with respect to supply of live animals. The supply function within the region was again considered to be inelastic; when the over-all supply of animals facing the plant within any given region was considered, the supply function was upward sloping because of the transportation costs. No consideration has been made, however, of the production costs of the animals in feedlots or on farms. Here again if the supply function within a region is not inelastic, the equilibrium picture could easily be altered.

Any model which aggregrates areas into representative regions must also make some designation with respect to intra-regional assembly and distribution costs. For purposes of this study these costs were assumed to be equal for all regions, and therefore were not considered in the programming problems; they would have simply added a set increment to the costs used, and would not have altered the solution. But with varying density rates of both live animals and consumers, the assembly and distribution costs probably would vary among regions and thus modify the solution presented.

The use of representative origins and destinations also precludes the selection of other locations in a particular area which might be equally as appropriate
as the one selected. To eliminate this limitation, however, would mean using a model with an extremely large number of possible sites and would require more data than is possible with the resources available.
Finally, the analysis presented here is of a static nature. Prices and costs are for 1960, but in industry conditions are subject to continual change, and as conditions change so may the equilibrium pattern indicated for one particular time period. Further, the above analysis deals with one year, whereas in actuality the nature of the slaughtering business is such that seasonal factors do not yield a continuous flow of live animals and dressed beef but, rather, show ups and downs within the year both in supply and demand.

## IMPLICATIONS

The plants slaughtering more than 300,000 pounds live weight annually (both specialized and diversified plants) in 1960 were scattered over the state (U.S. Agr. Mktg. Serv., 1960). The large clusters of plants around the Los Angeles and Bay areas resulted in most of the processing being done at those two locations. In 1960, some 1,292,000 head were slaughtered in Los Angeles County, while 244,000 head were processed in the Bay area (Calif. Crop and Lustk. Rptg. Serv., 1962). ${ }^{30}$ These levels in dressed weight are equivalent to 808.8 million pounds and 152.7 million pounds, respectively.

The analysis presented above indicates that over-all costs of processing and shipping could be reduced by centralizing slaughter into 12 locations with approximately 16 plants (assumes multiples of largest plant when supply exceeds capacity of 1 plant). Regions with 1 plant include $6,12,14,18,21,22$, 25,26 , and 27 . Region 15 and 32 have 2 plants each, while region 28 has 3 plants. While under the programming solution most of the state's slaughter

[^26]would be performed in the Los Angeles and Bay areas, some change is indicated from present patterns. The slaughter in Los Angeles County in 1960 was 187 per cent of that indicated by the programming solution, while the processing in the Bay area was approximately 46 per cent of the equilibrium solution. The Bay area in this case includes a smaller number than was included in the transhipment problem; therefore, the actual slaughter figures given here are likely too conservative.

In addition to a reduction in number of plants, the programming solution indicates a reduction in total costs by shifting more to the on-the-rail type of slaughtering. Out of the sixteen plants which could handle the state's slaughtering needs, only five would be of the bedtype category. These plants would be at Red Bluff, Stockton, Salinas, Ventura, and Santa Barbara (using points of origin as representative locations). The initial four would fall in the two-bed category (although not all would use the kill floor to full capacity) while Santa Barbara requirements could be met by a one-bed plant. Plant requirements in other regions would vary from one with a kill rate of slightly less than 60 head per hour (Sacramento), to multiple plant needs such as three plants in the Los Angeles region killing about 120 head per hour apiece.

The number of plants actually slaughtering cattle in California surpasses the number indicated by the programming solution, and most of them are smaller than those found in the final programming solution. A number of reasons exist for this situation. Many of the California plants are old plants. Buildings and equipment may be completely depreciated, resulting in lower total costs than the levels estimated for plants in this study. Business arrangements with feedlots or retail outlets may reduce buying and selling costs, thus enabling a plant to maintain operations which would be of higher cost otherwise. Additionally, diversified slaughtering
in some areas faced with seasonal shifts in supply of animals may provide a uniform operation during the entire year even though the quantity of cattle, when considered by itself, may be relatively small. Despite the existence of many small plants in the state, the number of diversified plants appears to be declining.

Although published data concerning the size distribution of plants in California are limited, there has been evidence of a decrease in some particular categories of plants. The United States Department of Agriculture classifies plants as "medium" (annual output of 300,000 pounds to $2,000,000$ pounds live weight) and "large" (annual output of more than $2,000,000$ pounds live weight). The nonfederally inspected medium size plants (all species) declined by 50 per cent from 1955 to 1960 with 22 and 11 plants in those years, respectively. The large nonfederally inspected plants decreased from 53 to 48 in the same period. The number of federally inspected plants (which in general tend to fall in the "large" classification) increased from 56 to 59 (U.S. Agr. Mktg. Serv., 1960).

Considering the slaughtering regions indicated by the transhipment solution, it seems evident that slaughtering activity is tied closely to the feeding of cattle. Eight of the regions designated as slaughtering areas are among the top nine regions in the state with respect to the estimated number of cattle marketed from feedlots in 1960. Because of the uncertainty surrounding the supplies of live animals packers have engaged in custom feeding of cattle in feedlots, or have turned to the actual operation of integrated feedlots to assure themselves of a steady supply of slaughter animals. Therefore, it would seem that the future pattern of slaughtering location would be tied closely to feedlot activity.

With the apparent bond between cattle feeding and slaughtering, continued urban expansion may create a pressure to move feeding activity from
heavily populated areas such as the Los Angeles and Bay areas. In this event, a decentralization of slaughtering might
occur as a result of the importance of transportation costs of shipping live animals long distances.

## ACKNOWLEDGMENTS

This analysis was made as a contributing study to Western Regional Research Project WM-37, "Economics of Transportation of Livestock and Meat in the Western Region," under the authority of the Agricultural Marketing Act of 1946. The authors are indebted to James N. Boles of the Giannini Foundation for initially sug-
gesting the applicability of the transhipment model to this problem. Appreciation is also expressed to H. O. Carter, B. C. French, I. J. Hoch, G. M. Kuznets, all of the Giannini Foundation, and to G. G. Judge of the University of Illinois, for their constructive comments and criticisms in review of this manuscript.

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[^0]:    ${ }^{1}$ Submitted for publication April 15, 1964.
    ${ }^{2}$ See "Literature Cited" for publications referred to in the text by author and date.

[^1]:    ${ }^{3}$ A similar derivation, without consideration of assembly or distribution costs can be found in Patinkin (1947). See also a comment by Leontief (1947). Patinkin studies cost minimization for a multiple-plant firm and discusses conditions whereby equating marginal costs may not result in a minimum-that is, the second order conditions are not met. Specific examples of such conditions will be discussed later in this paper.

[^2]:    ${ }^{4}$ For development of maxima and minima criteria, see Henderson and Quandt (1958), pp. 272-274.

[^3]:    ${ }^{5}$ Within region raw-product assembly costs are taken to equal zero in this development. However, as production in a region is increased, these assembly costs would increase. This continuous location model has been discussed by French (1960) and Henry and Seagraves (1960).
    ${ }^{6}$ Capacity may be considered either technically or economically. Technical capacity in this case refers to the maximum rate of output possible for the plant regardless of costs, and is bounded by the technical nature of the plant and equipment. The economic capacity indicates the least average cost point of operation. Capacity or size in the cost-volume relationships will be assumed to be one of economically efficient output for a plant operating one 8 -hour shift per day as opposed to maximum technical output.

[^4]:    ${ }^{7}$ Actually, transportation rates do show economies of scale as the quantity shipped increases. In this case, however, the shipments of raw product and final good were in sufficient quantity to qualify for the lowest rate, and thus, in effect, the transportation rate can be considered fixed.

[^5]:    ${ }^{8}$ The general nature of a factor-product model of spatial equilibrium may be seen in the study by Schrader and King (1962). The model presented here is based on an unpublished paper by Judge. (1962).
    ${ }^{9}$ In the following empirical analysis, an average value of $a_{i}$ and an average live weight per head for the entire state have been used in estimating meat supplies. It is recognized, however, that the dressing percentage and live weight may vary somewhat between regions, particularly if the regions are heavily populated with dairy animals (or conversely with heavy-fed cattle). Because of the heterogeneous composition of cattle supplies in most of the regions, it was assumed that errors in estimation of meat supplies for the regions resulting from this specification would be negligible.

[^6]:    ${ }^{10}$ There are several studies in which this problem of plant location with economies of scale is considered in relation to assembly costs or distribution costs, namely Simmons (1959), Stollsteimer (1961), Mathia (1962), Olson (1959), Henry and Seagraves (1960), French (1960), Williamson (1962). King and Henry (1959) have written a useful article reviewing the use of the transportation model with modifications.
    ${ }^{11}$ For a similar procedure see Giaever and Seagraves (1960).

[^7]:    ${ }^{12}$ In this example, each source region is also a demand region and vice versa, resulting in a square submatrices and a square over-all matrix. This is not essential. In fact, in the following analysis the submatrices are not square. If the latter is the case, this general three-region presentation can be modified to include nonsquare submatrices.

[^8]:    * Difference between total slaughter and number of animals a vailable from above categories. Composed mainly of dairy bulls, steers on farms.

    Source: California Crop and Livestock Reporting Service, California Annual Livestock Report, Summary for 1960. Sacramento (1961).

[^9]:    ${ }^{13}$ For proof of the duality of the transportation model, see Dorfman, Samuelson, and Solow (1958), pp. 122-129.

[^10]:    ${ }^{14}$ Unless otherwise specified, the figures on state supplies of animals were all derived or taken directly from California Crop and Livestock Reporting Service (1961), pp. 7-13.

[^11]:    * Includes feedlot marketings, dairy animals, cull beef, and grass-fed animals.
    $\dagger$ Number of head times 626 pounds (includes by-products).
    $\ddagger$ Total state supply divided by state population of $15,717,204$, then multiplied by county population level for 1960 .

[^12]:    ${ }^{15}$ Derived by multiplying the product of the number of animals slaughtered in 1960 and 1,026 pounds by 57.3 per cent.
    ${ }^{16}$ State and county population figures taken from Bureau of the Census (1961b).

[^13]:    ${ }^{17}$ Although an attempt was made to combine counties into more uniform areas, such grouping is admittedly arbitrary.

[^14]:    ${ }^{18}$ Inspection of the rate table for meat issued by the Public Utilities Commission reveals that for distances over 800 miles the cost of transportation increases at the relatively constant rate of 5 cents per 100 pounds per 50 miles additional travel.
    ${ }^{19}$ Although the Los Angeles rate is the lowest processing rate in the state, its assignment to out-of-state meat may bias the costs of inshipments upward. Correspondence between R. L. Shutt, Assistant Research Director of the United Packinghouse Workers of America, A.F.L.-C.I.O., Chicago, and the authors indicate that labor costs in Los Angeles and San Francisco areas run somewhat higher than for the metropolitan areas elsewhere in the country.

[^15]:    ${ }^{20}$ For a detailed description of the method of synthesizing costs, see L. L. Sammet (1958).

[^16]:    ${ }^{21}$ Designed capacity is defined as maximum number of cattle which can be slaughtered per hour by a given plant．
    ${ }^{22}$ In order to escape the problems of cost differentials among various areas of the state in the initial part of the analysis，the cost rates used were for the Los Angeles area，the principal slaugh－ tering area of California．Plants were assumed to operate one 8 －hour shift a day， 5 days a week， and 52 weeks a year with 8 paid holidays during this period．

[^17]:    - Includes full-time man working in the basement at regular pay.

    Source: Logan, Samuel H., and Gordon A. King, Economies of Scale in Beef Slaughter Plants, Giannini Foundation Research Report 260, University of California, Berkeley (1962).

[^18]:    * The annual slaughter figures used to calculate the cost per head (preceded by the hourly kill rate in parentheses) are as follows: (17) 32,004 ; ( 35 ) 66,024 ; ( 50 ) 94,500 ; (20) 37,800 ; (40) 75,600 ; (60) 113,400 ; (75) 141,624 ; (120) 226,800.

    Source: Logan, Samuel H., and Gordon A. King, Economies of Scale in Beef Slaughter Plants, Giannini Foundation Research Report 260, University of California, Berkeley (1962).

[^19]:    ${ }^{23}$ Working Agreement and Wage Scale, October 1, 1959, by and between the Butchers' Union, Local No. 563, A.F.L.-C.I.O., and Independent Meat Packers of Los Angeles County, California. Also, Swift and Company Master Agreement with United Packinghouse Workers of America ( $A F L-C I O$ ), covering period October 23, 1959 to September 1, 1961.

[^20]:    * Includes installation charge, freight costs, and 4 per cent sales tax

    Source: Logan, Samuel H., and Gordon A. King, Economies of Scale in Beef Slaughter Plants, Giannini Foundation Research Report 260, University of California, Berkeley (1962).

[^21]:    ${ }^{24}$ Initially a function was estimated which had a negative intercept. Since this is unacceptable logically the above function was used.
    ${ }_{25}$ Electricial rates were taken from Schedule A-7 General Service of Southern California Edison Company, Los Angeles, effective January 15, 1958. Water rates were taken from Ordinance $A p$ proving New Water Rates Fixed by the Department of Water and Power, City of Los Angeles, effective on bills dated on and after November 1, 1959. Section 2 gas rates came from Schedule G-50 of the Southern California Gas Company, Los Angeles.
    ${ }^{28}$ While the quantity of labor in number of men is a variable input, such variation is made over time periods of greater than 1 week, as the union contract guarantees payment for a 40 -hour week.

[^22]:    ${ }^{27}$ Electrical rates for northern California were taken from Pacific Gas and Electric Company Schedule P-3, General Power-Maximum Demand Basis-Alternating Current, effective November 15, 1957.
    ${ }^{28}$ Northern California gas rates were taken from Pacific Gas and Electric Company Schedule No. G-50, Interruptable Natural Gas Service, effective August 15, 1960. For Los Angeles gas and electric rates used in the study, see Logan and King (1962), pp. 85, 87.

[^23]:    * Costs are reduced by further adjustments: Stockton (region 14) sends dressed beef rather than animals to Bay area (region 15), while Bakersfield (region 25) sends meat rather than animals to Los Angeles (region 28). As a result, Stockton slaughters 34 million pounds, Bay area 285, Bakersfield 145, and Los Angeles 432.

[^24]:    ${ }^{20}$ In the final solution the shipment pattern from Stockton to Oakland, and from Bakersfield to Los Angeles, was altered to dressed beef rather than live animals in order to retain lower

[^25]:    slaughtering costs in the surplus areas. As a result the prices shown for the live animals in Stockton are slightly lower than those that could be obtained by shipping to the Bay area; however, if the latter pattern were used the cost of slaughtering in Stockton would increase and the price of cattle sold there would be reduced. These increased costs would be more than the reduction in revenue resulting from selling all animals in Stockton and shipping meat to Oakland. For Bakersfield, transportation and slaughtering costs are such that the price to producers is the same (live weight basis) selling all animals in Bakersfield as it would be to sell them in Los Angeles. In addition, slaughtering costs in Bakersfield are lowered.

[^26]:    ${ }^{30}$ Bay area includes Alameda, Contra Costra, Santa Clara, San Mateo, and San Francisco counties.

